Limited (Energy) Supply, Monetary Policy, and Sunspots*

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Abstract

In a simple New Keynesian open economy setting, we analyze how local input shortages influence policy transmission and equilibrium determinacy. Shortages increase the elasticity of the local price of the scarce factor to domestic economic activity, affecting the cyclicality of marginal costs and incomes. As a result, the slope of both the Phillips and the IS curve is altered, crucially influencing monetary and fiscal policy transmission. These changes are affected by factor ownership and propensities to consume. Theoretically, shortages can also raise the risk of self-fulfilling fluctuations if a rising price of the constrained factor boosts incomes for agents with high propensities to consume. We illustrate these channels for the 2022 German energy crisis.

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1 Introduction

The international macroeconomic environment is increasingly affected by shortages of inputs. The underlying causes can be manifold, ranging from supply chain disruptions driven by natural disasters to policy decisions motivated by geopolitics. What these shortages have in common is a potentially substantial change in the sensitivity of prices to local demand conditions, as supply might no longer be abundant at a given price – even from the perspective of a small open economy and for imported inputs. As the local price feeds back to demand and cost conditions, shortages can have profound implications for stabilization policy.

This paper examines the implications of such supply constraints for monetary transmission and macroeconomic stability in a New Keynesian open economy model. Domestic production is assumed to use labor and another input factor. The income from the latter accrues to domestic households and to a foreign economy that demands domestic products in return. This core of the model is the same as in Blanchard and Galí (2009). We add liquidity constrained households, like in Bilbiie (2008), supply constraints for the input factor, and fiscal policy that shields households and firms from fluctuations in the input's price. In our setting supply constraints alter the cyclicality of the factor's price, which changes the slope of the Phillips curve and the cyclical distribution of income across households and countries, the demand side. Through both, supply constraints alter the effectiveness and distributional consequences of monetary and fiscal policy. In addition, macroeconomic stability might require a response to the factor's price movements because of the factor's central role in the distribution of income.

Before turning to a quantitative exploration, we use a simplified version of the model to derive with paper and pencil how different dimensions of the model shape the effect of a shortage of an input on aggregate supply and demand, monetary transmission, and equilibrium determinacy. These derivations assume that the potentially constrained factor is only used in production and that trade is balanced. These assumptions allow for a representation that has the same three equations as the textbook New Keynesian model: a Phillips curve, a dynamic IS equation, and a Taylor rule. We compare two supply regimes. In one, the factor is in abundant supply at a fixed price, a common assumption in the literature (for example, Blanchard and Galí, 2009). In the other regime, the factor's supply to the domestic economy is fixed. Instead, its price moves flexibly to clear the market. This deviation from the literature is simple but consequential.

Supply constraints tend to steepen the Phillips curve. This finding is true unless the price movements of the constrained factor are accompanied by strong wealth effects on domestic labor supply that arise on the back of redistribution between the domestic and the foreign economies. The impact of supply constraints on the IS curve and thus the aggregate demand relation depends on how the constraints affect the cyclical distribution of incomes. This in turn depends on the composition of the after-tax incomes of the model's different agents: Ricardian households (savers), hand-to-mouth households, and the foreign economy. If supply constraints render savers' incomes less procyclical – for example, because the firms they own face more procyclical energy costs – demand becomes more interest sensitive. The aggregate demand curve flattens in inflation-output space. In the extreme, we show that if supply constraints render the incomes of savers countercyclical altogether then such constraints raise the risk of indeterminacy. A response by monetary policy to inflation stronger than that prescribed by the Taylor principle would be required.[1]

In light of the fact that the distribution of income is crucial in determining the implications of supply constraints, we turn to a quantitative exploration. We calibrate the model to the German economy, associate the input factor with energy, and build a scenario reflecting the energy shortages that Germany witnessed in the run-up to and after the 2022 Russian invasion of Ukraine. Relative to the paper-and-pencil results, we allow for energy use also in consumption and allow trade not to balance period by period.

We first focus on how the effects of a monetary easing change once the supply constraints are in place. As Germany imports roughly 2/3 of its energy and uses a similar amount in production, energy supply constraints make a monetary easing both more inflationary and less effective at stimulating domestic demand, in line with a steeper Phillips curve and flatter aggregate demand curve. Taking the perspective of the "sacrifice ratio" this

[1] The "Taylor principle" states that determinacy is ensured if the central bank responds more than one to one to inflation – that is, if $\phi_\Pi > 1$ but is arbitrarily close to 1.

implies that monetary policy has to sacrifice notably less real activity and consumption in order to bring down core (or headline) inflation by a certain amount once the energy supply constraint binds. For the same reasons, we find that the energy supply constraint means that a fiscal transfer to hand-to-mouth households crowds in domestic consumption by less and, again, is more inflationary.

We also entertain one, perhaps more subjective, interpretation of the economic environment in the energy crisis. In this, energy price related fiscal interventions shield households and firms from energy price increases and a tight labor market gives rise to more flexible wages. We show that in this environment, if monetary policy looks through energy price movements by reacting only to core inflation, a sunspot equilibrium can arise in which higher (lower) energy prices go hand in hand with higher (lower) economic activity.

The intuition is as follows. Suppose that households and firms hold a *non-fundamental* (sunspot) belief of high demand for domestic products. High demand will have to be met by high output. Because energy supply is fixed, higher output requires hours worked to rise. Wages rise, and the energy price increases disproportionately such that firms substitute labor and energy. Price rigidities mean that firms cannot fully pass these costs on to consumers. If savers receive the profit income, their share of income falls. Foreign's income instead rises if Foreign owns the energy supply. Meanwhile, hand-to-mouth households' (labor) income increases as well on the back of flexible wages. Provided the foreign economy has a reasonably large marginal propensity to import goods, aggregate demand (domestic plus external) can therefore be high, supporting the non-fundamental beliefs. In this feedback loop, high energy prices are a *symptom* of high demand meeting supply constraints. Thus, the key policy implication is that monetary and fiscal policy can avoid the loop if they lean sufficiently strongly against demand. In our crisis scenario, a monetary response to headline, rather than core, inflation at conventional strengths would already be sufficient to ensure determinacy. Whether the risk of indeterminacy mattered in practice, thus, depends on one's view of the scenario and on whether or not the central bank saw through energy price movements or responded to them.

The rest of the paper is structured as follows. We review the literature next. Section 2 presents the model. Section 3 discusses our pencil-and-paper analysis. Section 4 illustrates the channels for the German energy crisis. A final section concludes.

Related literature. We analyze how supply constraints shape monetary policy transmission and equilibrium determinacy, both in theory and in an application to the German energy crisis of recent years. We pay particular attention to how these supply constraints affect aggregate outcomes through their effect on the distribution of income.

Ours, of course, is not the first paper to study the business cycle implications of supply constraints. Álvarez-Lois (2006) and Fagnart, Licandro and Portier (1999), and Kuhn and George (2019) analyze the role of capacity constraints in the propagation of aggregate shocks. Boehm and Pandalai-Nayar (2022) provide empirical evidence for sizable convexities in the supply curves of U.S. industries due to capacity constraints. Balleer and Noeller (2023) argue that constraints on inputs empirically shape the transmission of monetary policy. Comin, Johnson and Jones (2023) analyze the role of occasionally binding capacity constraints with an emphasis on supply chains. Relative to the above literature, we focus on how supply constraints affect aggregate demand and supply through the redistribution of resources across households and countries.

While our theory applies to any supply-constrained input, our quantitative application interprets the constrained factor as energy. In our calibrated model, the effect of an exogenous shock to the energy price is in line with empirical estimates of the propagation of exogenous fundamental energy shocks – provided, for example, by Baumeister and Hamilton (2019), Blanchard and Galí (2009) and Blanchard and Riggi (2013), and Känzig (2021). Such energy shocks are not the focus of our paper, however. We, instead, wish to analyze how supply constraints to energy shape the transmission of monetary and fiscal policy or may allow for the emergence of sunspot-driven fluctuations.

In terms of modeling, we rely on Blanchard and Galí (2009) and Blanchard and Riggi (2013), who point to the structural features that shape the response to fundamental shocks to the price of, in their case, oil. Meanwhile, the modeling of heterogeneity and its cyclical consequences builds on the closed economy results in Bilbiie (2008) and Bilbiie (2021). Olivi, Sterk and Xhani (2022) and Känzig (2023) have analyzed the distributional effects of *exogenous* energy price changes. All of these papers consider an environment of abundant energy supply, whereas we focus on the effect of supply constraints. Datta et al. (2021) focus on energy shock transmission amid the interest rate lower bound from which we abstract. Pieroni (2023) and Auclert et al. (2023) provide assessments of fundamental energy shocks in heterogeneous household New Keynesian models. Kharroubi and Smets (2023) study optimal fiscal policy when there are cuts in energy supply in a two-household setting with flexible prices. Relative to all these papers, we provide closed-form expressions for how supply constraints affect both the Phillips curve and the IS curve. And we highlight that supply constraints can support sunspot fluctuations, particularly, if fiscal policy works to shield households and firms from the effects of the price movements.

Our simulations assume the usual policy prescription of central banks to "see through" price movements of goods that have flexible prices, a prescription backed by ample theory.[2] With supply constraints, the price of the constrained good can be as much an indication of the state of supply as of demand. If high demand is associated with a redistribution of income to agents with higher marginal propensities to consume, fiscal policy, monetary policy, or both may need to be less accommodative to avoid self-fulfilling fluctuations. These results do not rely on fiscal policy turning active in the sense of the fiscal theory of the price level (see, for example, Leeper (1991), Schmitt-Grohé and Uribe (2007), or Kumhof, Nunes and Yakadina (2010)). Rather, they are derived under a balanced fiscal budget. In this setting indeterminacy can arise when the supply constraints induce an inversion of the relationship between aggregate demand and the *ex ante* real interest rate.[3] Monetary policy remains free, though, to implement the Taylor principle.[4] This freedom sets our work apart from papers that study non-conventional transmission at the effective lower bound: under determinacy (see, for example, Eggertsson (2011)), or indeterminacy (see, for example, Mertens and Ravn (2014)).

[2] On the positive side, see, for example, Carlstrom, Fuerst and Ghironi (2006) and Airaudo and Zanna (2012). On the normative side, see, for example, Aoki (2001) and Bodenstein, Erceg and Guerrieri (2008).

[3] Bilbiie (2008) studies the determinacy implications of an inverted IS curve in a closed economy.

[4] It would be interesting to see how other mechanisms that invalidate the Taylor principle (see, for example, Kara and Yates (2021) and the references therein) interact with supply constraints.

2 Model

Consider the following infinite horizon model of two countries: Home and Foreign. The focus is on the Home economy and a generic, non-storable good E, the supply of which can be constrained. The label E is chosen to indicate that the good is "essential" to Home in that it is both consumed and used as an input in production.[5] Households in Home are heterogeneous, as in Bilbiie (2008). Firms in Home are subject to nominal rigidity. Foreign is not modeled in detail. It serves as a source of the essential good, which it exports in exchange for goods that firms in Home produce, as in Blanchard and Galí (2009). We keep the exposition concise. Appendix A provides additional details. Appendix B lists all the model equations.

2.1 Households in Home

Home is populated by two types of representative, infinitely lived households: a mass $\lambda \in [0, 1)$ of hand-to-mouth households, H, and a mass $1 - \lambda$ of saver households, S. Hand-to-mouth households do not have access to financial markets. They consume all of their income each period. Savers, however, optimize intertemporally. They can save in liquid, risk-free nominal bonds, the rate of return on which the central bank controls. Preferences in the two groups are identical. For any $i \in \{H, S\}$, they are given by:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right] \right\}, \text{ with } \beta \in (0,1),\ \sigma > 0,\ \chi > 0, \text{ and } \varphi \geq 0.$$

Here, $\mathbb{E}_t$ marks the expectations conditional on period-t information; $N_{i,t}$ marks hours worked. The consumption index $C_{i,t}$ is given by the following expression:

$$C_{i,t} = \left[\gamma^{\frac{1}{\eta}} \left(C_{i,E,t} - \bar{e} \right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{i,G,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{1}$$

Households thus derive utility from consuming the E-good and another good marked G. $\bar{e} \geq 0$ is a subsistence level for consumption of the E-good, $\gamma \in (0, 1)$ the good's preference

[5] E is a fitting label for other reasons, too. In the current paper, the supply of E is an "endowment," E may be owned "externally," and, in the application later, E will be associated with "energy."

weight, and $\eta > 0$ the elasticity of substitution between the two goods.

Let $P^c_{E,t}$ and $P_{G,t}$ mark the consumer prices for the E-good and G-good, respectively. Also, define price index $P_t = \left[\gamma {P^c_{E,t}}^{1-\eta} + (1-\gamma){P_{G,t}}^{1-\eta}\right]^{1/1-\eta}$. With this, in nominal terms, the hand-to-mouth household's budget constraint is given by the following:

$$P^c_{E,t} C_{H,E,t} + P_{G,t} C_{H,G,t} = (1+\tau^w) W_t N_{H,t} + P_t T_{H,t}. \tag{2}$$

The left-hand side features the household's consumption expenditures for the two types of goods, and the right-hand side includes the household's income. $(1+\tau^w)W_t$ is the nominal wage adjusted for a wage subsidy. $T_{H,t}$ marks real lump-sum transfers to hand-to-mouth households. The saver's budget constraint, in turn, is given by the following:

$$P^c_{E,t} C_{S,E,t} + P_{G,t} C_{S,G,t} + B_t/(1-\lambda) = (1+\tau^w) W_t N_{S,t} + P_t T_{S,t} + R_{t-1} B_{t-1}/(1-\lambda). \tag{3}$$

Savers have access to the bond market. Bonds pay a nominal gross return of R_t in period $t+1$. $B_t/(1-\lambda)$ marks the saver's period-t expenditure for bonds.[6] Budget constraints (2) and (3) do not mention firms' profits or the proceeds from ownership of the E-good. For tractability, the exposition subsumes such cash flows in the transfers $T_{H,t}$ and $T_{S,t}$. Section 2.4.2 will discuss the transfers and the implied ownership structure.

Both types of households allocate consumption optimally within the period, taking prices as given. Consumption demand for the two types of goods is thus given by the following:

$$C_{i,E,t} = \bar{e} + \gamma \left(P^c_{E,t}/P_t\right)^{-\eta} C_{i,t}, \quad \text{and} \quad C_{i,G,t} = (1-\gamma)\left(P_{G,t}/P_t\right)^{-\eta} C_{i,t}, \quad i \in \{H, S\}.$$

Hand-to-mouth households' consumption expenditures simply equal their income. Savers instead allocate consumption optimally over time. Defining headline inflation as $\Pi_t := P_t/P_{t-1}$, the associated Euler equation is the following:

$$C^{-\sigma}_{S,t} = \beta\, \mathbb{E}_t\{C^{-\sigma}_{S,t+1} R_t/\Pi_{t+1}\}.$$

[6] Our notation already anticipates that, in equilibrium, the only counterparty to saver households for savings will be Foreign.

2.2 The labor market in Home

The labor market may be characterized by rigid nominal wages. To avoid unnecessarily cumbersome notation, we state the corresponding equilibrium relationships here. Appendix A.1 provides the microfoundation, following Colciago (2011). Under nominal wage rigidity, all households supply the same amount of labor ($N_{H,t} = N_{S,t}$) and the wage, with nominal wage inflation defined as $\Pi_{W,t} := W_t/W_{t-1}$, moves according to:

$$\Pi_{W,t}(\Pi_{W,t}-1) = \frac{\varepsilon^w}{\psi^w}\left(\frac{\chi N_t^{\varphi}}{\lambda C_{H,t}^{-\sigma} + (1-\lambda)C_{S,t}^{-\sigma}} - (1+\tau^w)\frac{\varepsilon^w - 1}{\varepsilon^w}\frac{W_t}{P_t}\right) \\ + \beta\,\mathbb{E}_t\left\{\frac{\lambda C_{H,t+1}^{-\sigma} + (1-\lambda)C_{S,t+1}^{-\sigma}}{\lambda C_{H,t}^{-\sigma} + (1-\lambda)C_{S,t}^{-\sigma}}\,\Pi_{W,t+1}(\Pi_{W,t+1}-1)\,\frac{N_{t+1}}{N_t}\right\}.$$

$\varepsilon^w > 1$ is the elasticity of substitution between different varieties of inputs of "labor services." $\psi^w > 0$ measures the Rotemberg-type wage adjustment costs. N_t denotes aggregate labor supply.

In the case without wage rigidities, we instead assume that households choose their labor supply flexibly and without market power (or an accordingly set τ^w), meaning that $W_t/P_t = \chi C_{i,t}^{\sigma} N_{i,t}^{\varphi}$ for each $i \in \{H, S\}$.

2.3 Production in Home

There is a unit mass of producers that is indexed by $j \in [0,1]$. Each produces one variety of a differentiated good using the essential good and labor as inputs. Production follows

$$y_{G,t}(j) = \left[\alpha E_t(j)^{\frac{\theta-1}{\theta}} + (1-\alpha)N_t(j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Here, $\alpha \in (0,1)$ governs input shares, and $\theta \in (0,1)$ is the elasticity of substitution of the two inputs. Producers j sell their differentiated goods under monopolistic competition, to a competitive retailer that bundles goods into the final G-good according to the following:

$$Y_{G,t} = \left[\int_0^1 y_{G,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \text{ with } \epsilon > 1.$$

Differentiated goods producers are subject to quadratic price adjustment costs. Appendix A.2 presents the price-setting problem in detail. The producers' first-order conditions give rise to a New Keynesian Phillips curve in producer price inflation, with the latter defined as $\Pi_{G,t} := P_{G,t}/P_{G,t-1}$. This curve is given by the following expression:

$$\begin{aligned}\Pi_{G,t}(\Pi_{G,t}-1) &= \frac{\varepsilon}{\psi}\left(\frac{\Lambda_t}{p_{G,t}} - (1+\tau^y)\frac{(\varepsilon-1)}{\varepsilon}\right) \\ &\quad + \beta\,\mathbb{E}_t\left\{\left(\frac{C_{S,t+1}}{C_{S,t}}\right)^{-\sigma}\Pi_{G,t+1}(\Pi_{G,t+1}-1)\,\frac{Y_{G,t+1}}{Y_{G,t}}\,\frac{p_{G,t+1}}{p_{G,t}}\right\}.\end{aligned}$$

Here, τ^y is a sales subsidy. $\psi > 0$ measures the Rotemberg-type price adjustment costs. Λ_t marks real marginal costs in consumption units and $p_{G,t} := P_{G,t}/P_t$ the real goods price.[7] The relevant marginal costs thus are expressed in units of the produced good. Letting $P^f_{E,t}$ denote the price for the E-good that *firms* pay, the optimal choice of factor inputs implies $\alpha\, W_t/P^f_{E,t} = (1-\alpha)\,(E_t/N_t)^{1/\theta}$. Real marginal costs follow

$$\Lambda_t = \left[\alpha^\theta\big(P^f_{E,t}/P_t\big)^{1-\theta} + (1-\alpha)^\theta\big(W_t/P_t\big)^{1-\theta}\right]^{1/(1-\theta)}.$$

Last, in equilibrium, the firm sector's nominal profits are given by the following equation:[8]

$$P_tD_t = (1+\tau^y)P_{G,t}Y_{G,t} - W_tN_t - P^f_{E,t}E_t.$$

2.4 Fiscal policy in Home

The government runs a balanced budget. Transfers to savers, $T_{S,t}$, are the balancing item. We consider this assumption to be the most innocuous on government funding over the business cycle that one can make. The government budget constraint is given by:

$$P_tD_t + \iota\, P_{E,t}\,\xi_{E,t} = \tau^y P_{G,t}Y_{G,t} + \tau^w W_tN_t + (P_{E,t} - P^c_{E,t})C_{E,t} + (P_{E,t} - P^f_{E,t})E_t + P_tT_t.$$

[7] We apply a similar notation to other real prices later, for example, $p^c_{E,t} := P^c_{E,t}/P_t$.

[8] Below, we work with the model after linearizing around a zero-inflation steady state. To keep expressions simple here, we therefore decided not to display the price and wage adjustment costs in the profits, in households' incomes, or in the resource constraint as they are zero to first order.

The left-hand side features the government's sources of revenue. The government receives all the dividends in the Home economy. In addition, the local economy also is endowed with a share $\iota \in [0,1]$ of the total supply, $\xi_{E,t}$, of the E-good. Thus, the revenues from selling this share of the good also accrue to the government. The right-hand side of the government budget constraint features the government's expenditures. The government subsidizes production and employment. It also may provide subsidies to the use of the E-good in consumption, with $C_{E,t} := \lambda C_{E,H,t} + (1-\lambda)C_{E,S,t}$, or production, E_t. In this case, the government bears the difference between the wholesale price $P_{E,t}$ that the owners of the E-good receive and the price charged to consumers or firms. Lastly, the government makes lump-sum transfers, with $T_t := \lambda T_{H,t} + (1-\lambda)T_{S,t}$.

2.4.1 Subsidies for the E-good

If the supply of the E-good is constrained, its wholesale price fluctuates. We may allow the government to lean against the resulting fluctuations in the price that consumers and producers face. We parameterize the degree of this policy response by parameters $\tau_E^c \in [0,1]$ and $\tau_E^f \in [0,1]$, assuming the government ensures that:

$$\log(p_{E,t}^k/p_E) = (1-\tau_E^k)\log(p_{E,t}/p_E), \quad k \in \{c, f\}.$$

2.4.2 Transfers and ownership of cash flows

We assume that the transfers to all hand-to-mouth households are given by:

$$\lambda P_t T_{H,t} = P_t \overline{T}_H + \nu\big(P_t D_t - \tau^y P_{G,t} Y_{G,t}\big) + \iota\,\vartheta\, P_{E,t}\xi_{E,t} - \lambda \tau^w W_t N_{H,t} + P_t \zeta_t.$$

The transfers comprise a constant term, $\overline{T}_H$. In addition, the transfers reflect a claim to firms' profits (net of production subsidies). If $\nu < \lambda$, a hand-to-mouth household receives a smaller share of the profit income than a saver household – $\nu \in [0,1]$. Meanwhile, hand-to-mouth households receive a share $\vartheta \in [0,1]$ of Home's income from its endowment with the E-good. Note that ν and ϑ implicitly define how much of the respective streams of income hand-to-mouth households "own" (and savers do not). The government charges the households their respective share of the costs of the labor subsidy. Last, ζ_t is a zero-mean transfer shock that redistributes from savers to hand-to-mouth households.

2.5 Monetary policy in Home

The central bank controls the gross nominal interest rate R_t according to the Taylor rule

$$\log(R_t/R) = \phi_\Pi \cdot \log(\Pi_{G,t}/\Pi_G) + \upsilon_t, \text{ where } \quad \phi_\Pi \geq 0,$$

and υ_t is a monetary shock. A common prescription for the optimal response of monetary policy to relative price changes is for the central bank to focus on the inflation rates of those goods or services that are subject to nominal rigidities; see, for example, Aoki (2001). This means focusing on the inflation rate associated with the G-goods.

2.6 International trade and Foreign demand

Foreign matters as a source of supply of the E-good to Home and as a source of demand for goods produced in Home – the G-goods. Foreign's budget constraint (expressed in units of Home's currency) is the following:

$$P_{G,t}Y_t^* = P_{G,t}X_{G,t} - [B_t - R_{t-1}B_{t-1}].$$

On the left-hand side, $P_{G,t}Y_t^* := (1-\iota)\,\xi_{E,t}\,P_{E,t}$ denotes Foreign's current income from selling its share $(1-\iota)$ of the total supply of the E-good. The right-hand side marks Foreign's expenditures for imports (Home's exports of goods to Foreign, $X_{G,t}$) and the accumulation of net foreign assets (nominal bonds issued in Home's currency). This formulation reflects the fact that, in equilibrium, the savings of Home's savers must be mirrored in net foreign liabilities of Foreign. We do not seek to construct a more detailed model of the Foreign economy. Instead, we focus directly on Foreign's propensity to consume (MPC) out of any windfall gains associated with an increase in the price $P_{E,t}$.[9]

[9] What we label Foreign's "MPC" is identical to Foreign's marginal propensity to demand Home's goods when Foreign's income rises. We continue to use the familiar term "MPC."

Let $-b_t := -B_t/P_t$ denote Foreign's external asset position. We parameterize Foreign's demand for goods produced in Home as:

$$\log(X_{G,t}/X_G) = \mu_{F,1} \cdot \log(Y_t^*/Y^*) + \mu_{F,2} \cdot (-b_{t-1}/Y^*) \,.$$

Parameters $\mu_{F,1} \geq 0$ and $\mu_{F,2} \geq 0$ measure, respectively, Foreign's MPC out of current income and its MPC out of wealth.

2.7 Supply regimes for the E-good

We consider two regimes for the supply of the E-good. In the fixed-price regime, its real wholesale price, $p_{E,t}$, is constant. The good's supply, $\xi_{E,t}$ is perfectly elastic. Not having modeled the costs of producing the essential goods, this assumption serves as a point of reference, and it is in keeping with the literature, for example, Blanchard and Galí (2009). In the fixed-supply regime, however, the total supply to Home, $\xi_{E,t}$, is constant. In this case, the price of the essential good has to clear the market in Home.

2.8 Market clearing

In equilibrium, all markets clear. The labor market clears if $N_t = \lambda N_{H,t} + (1-\lambda) N_{S,t}$ – that is, labor demand of firms equals the different households' labor supply. The market for the E-good clears if $\xi_{E,t} = C_{E,t} + E_t$, that is, if supply meets the demand of households and firms. The market for domestic products clears if $Y_{G,t} = C_{G,t} + X_{G,t}$, that is, if production equals demand for consumption in Home and exports. Here $C_{G,t} := \lambda C_{H,G,t} + (1-\lambda) C_{S,G,t}$ represents total domestic consumption of the good.

3 Pencil-and-paper intuition

The departure of this paper from the literature is simple but consequential. We change the supply regime of a good that, among other possible uses, serves as an input to production – from a regime of elastic supply to a regime of fixed supply. This change alters the relationship between firms' marginal costs and output, and it changes the cyclicality of incomes. If agents with different MPCs rely on different sources of income, the supply regime also affects the distribution of income across agents, changing the interest elasticity of aggregate demand. Importantly, this changes the transmission of shocks and may alter the conditions for determinacy. Using a simplified version of the model from Section 2, the current section provides intuition by discussing a sequence of propositions. Appendix C presents all the derivations related to the model variant used in this section. Appendix D provides the proofs of the propositions.

3.1 Notation, parametric assumptions, and steady-state targets

Notation. The notation we will use is as follows. For a generic variable Z_t, let Z mark its steady-state value. Let $\widehat{Z}_t$ denote the percent deviation from the steady state. Below, we also rely heavily on the following notation: Let the convolute of parameters Γ_Z be defined such that, in equilibrium, $\widehat{Z}_t = \Gamma_Z \widehat{Y}_{G,t}$ – that is, Γ_Z marks the cyclical elasticity of $\widehat{Z}_t$ with respect to output $\widehat{Y}_{G,t}$. If we want to make explicit that Γ_Z depends on the supply regime, we add superscripts. Thus, Γ_Z^P marks the elasticity of $\widehat{Z}_t$ to output in the fixed-price regime and Γ_Z^Q denotes it in the fixed-supply regime ("Q" signifies a constrained quantity). The Γ_Z elasticities are general equilibrium elasticities. They are invariant to the shocks entertained in this section – that is, to monetary shocks or sunspot shocks. Another way to explain what Γ_Z is would be to describe it as the "cyclical sensitivity" of Z_t with respect to output.

Parametric assumptions. The aim of the current section is to provide closed-form intuition. Toward this aim, we abstract from the use of the E-good in consumption ($\gamma \to 0$, $\bar{e} \to 0$). We also abstract from subsidies for the E-good ($\tau_E^f = \tau_E^c = 0$) and from wage rigidity or wage markups. We set $1 + \tau^y = \epsilon/(\epsilon - 1)$ to remove price markups. If the E-good is not only owned domestically, $\iota < 1$, we assume that trade is balanced ($\mu_{F,1} = 1$) to avoid an endogenous state variable. And, only for the convenience of exposition, we let $\beta \to 1$. Throughout the section, the monetary policy shock v_t is white noise and we abstract from fiscal shocks, so $\zeta_t = 0$.

Steady-state targets. We consider a particular steady state that simplifies the exposition. We focus on a zero-inflation steady state ($\Pi_G = \Pi_W = 1$), and we assume that fiscal transfers to households in the steady state are such that consumption by the two types of households is symmetric in the steady state ($C_H = C_S$). The scaling parameter of the disutility of work χ sets $N_H = N_S = 1$. These assumptions imply that the slope of the Phillips curve is not affected by the distribution of income within the country. Finally, we normalize the steady-state output to $Y_G = 1$. The chosen value for τ^y then implies $E = 1$. For all that follows, we will look at a linear approximation of the equilibrium around the non-stochastic steady state. Appendix C.1.1 provides the nonlinear equilibrium conditions for the model variant used here, and Appendix C.1.2 provides the linearized equations. Appendix C.1.3 provides the steady state.

3.2 Determinacy and monetary transmission

Under the assumptions above, the linearized economy can be expressed in conventional form: a Phillips curve, a dynamic IS equation, and a Taylor rule.[10] Because producer price and consumer price inflation here coincide, the linearized Taylor rule is given by:

$$\widehat{R}_t = \phi_\Pi \widehat{\Pi}_t + v_t. \tag{4}$$

The Phillips curve and IS equation are given by the following textbook representation of the New Keynesian model:

$$\widehat{\Pi}_t = \mathbb{E}_t\{\widehat{\Pi}_{t+1}\} + \widetilde{\kappa}\ \widehat{Y}_{G,t} \qquad \text{and} \qquad \widehat{Y}_{G,t} = \mathbb{E}_t\{\widehat{Y}_{G,t+1}\} - \frac{1}{\widetilde{\sigma}}\left(\widehat{R}_t - \mathbb{E}_t\{\widehat{\Pi}_{t+1}\}\right). \tag{5}$$

Defining the "slope of the Phillips curve" as $\widetilde{\kappa}$ and the "slope of the IS curve" as $1/\widetilde{\sigma}$, the values of both slopes depend on the supply regime for the E-good.[11] Next, we spell out

[10] Appendix C.2 provides the steps involved.

[11] Note that the "slope of the IS curve" refers to the slope in output-real rate space. An "aggregate demand curve" arises from combining Taylor rule (4) and the IS curve in (5). It is usually drawn in inflation-output space, where its slope is inversely related to the "slope of the IS curve" as defined here.

how the slopes link to monetary transmission and local determinacy. Section 3.3 discusses how the supply regime affects the cyclicality of the E-good's price which is at the core of the results. Sections 3.4 and 3.5 thereafter discuss the economic determinants that govern each of these slopes.

Monetary transmission. Suppose the monetary response to inflation, ϕ_Π, is set to a value high enough to ensure determinacy. Then, (4) and (5) give that in equilibrium:

$$\widehat{\Pi}_t = -\frac{\widetilde{\kappa}}{\widetilde{\sigma} + \widetilde{\kappa}\phi_\Pi}\, v_t \qquad \text{and} \qquad \widehat{Y}_{G,t} = -\frac{1}{\widetilde{\sigma} + \widetilde{\kappa}\phi_\Pi}\, v_t. \tag{6}$$

Thus, to the extent that the supply regime affects $\widetilde{\kappa}$ and $\widetilde{\sigma}$, the supply regime also matters for the effectiveness of monetary stimulus and the trade offs that monetary policy faces. In other words, supply constraints affect the "sacrifice ratio," that is, how much real activity the central bank has to forfeit to reduce inflation by a certain amount.[12]

Determinacy. The following provides the conditions for determinacy that apply:

Proposition 1. *Consider the model of Section 2 and apply the assumptions listed in Section 3.1. Then, two cases summarize the conditions for local determinacy:*

1. *If $\widetilde{\sigma}$ and $\widetilde{\kappa}$ have the same sign, there is determinacy if and only if $\phi_\Pi > 1$.*
2. *If $\widetilde{\sigma}$ and $\widetilde{\kappa}$ have opposite signs, there is determinacy if and only if*

$$\phi_\Pi > \max\left(1, -4\,\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right). \tag{7}$$

Proof. The proof follows Woodford (2003, p. 670 ff) and is in Appendix D.1. □

The "Taylor principle" states that determinacy is ensured if the central bank responds more than one to one to inflation – that is, if $\phi_\Pi > 1$ but is arbitrarily close to 1. Inequality (7) shows that such a response may not be sufficient here, as the supply regime can affect $\widetilde{\kappa}$ and $\widetilde{\sigma}$. Namely, if $\widetilde{\sigma}/\widetilde{\kappa} < 0$, determinacy may require a notably stronger monetary

[12] By equation (6) and the definition of Γ_C as the cyclical sensitivity of consumption with respect to output, the sacrifice ratio with respect to consumption is given by $\Gamma_C/\widetilde{\kappa}$, for example, where both Γ_C and $\widetilde{\kappa}$ depend on the supply regime.

reaction: $\phi_\Pi >> 1$. Below, we provide conditions under which the supply constraint itself can render $\widetilde{\kappa} < 0$ or $\widetilde{\sigma} < 0$.

3.3 The cyclical sensitivity of the E-good's price

Shortages of an input factor change the sensitivity of prices to local demand conditions. As the local price feeds back to demand and cost conditions, shortages can have profound implications for stabilization policy. At the heart of our analysis thus lies a change of the sensitivity of the factor's price to demand. The fixed-price regime introduced in Section 2.7 means that quantities clear the market for the essential good. In this regime, the sensitivity of demand for the E-good to output is increasing (Γ_E^P greater) in the substitutability of the essential good (θ greater) and the sensitivity of marginal costs to output (Γ_Λ^P greater):

$$\Gamma_{p_E}^P = 0 \qquad \text{and} \qquad \Gamma_E^P = 1 + \theta\,\Gamma_\Lambda^P. \tag{8}$$

In the fixed-supply regime, however, the price must move to equalize demand for the E-good with a fixed supply ($\Gamma_E^Q = 0$). In this regime, the price is elastic to output, and the more so ($\Gamma_{p_E}^Q$ greater) the more sensitive marginal costs are to output (Γ_Λ^Q greater) and the less substitutable the essential good is (θ smaller). We have:

$$\Gamma_{p_E}^Q = \Gamma_\Lambda^Q + \frac{1}{\theta} \qquad \text{and} \qquad \Gamma_E^Q = 0. \tag{9}$$

3.4 Supply constraints and the slope of the Phillips curve

The supply constraint causes a change in the cyclicality of the E-good's price. The constraint also changes the cyclicality of marginal costs. This is important for inflation dynamics. Appendix C.2.5 derives that

$$\widetilde{\kappa} = \frac{\epsilon}{\psi}\,\Gamma_\Lambda. \tag{10}$$

Equation (10) means that the slope of the Phillips curve in (5) depends on the supply regime precisely through the elasticity of marginal costs to output, Γ_Λ. We summarize the effect of the supply regime on Γ_Λ in Proposition 2.

Proposition 2. *Consider the model of Section 2 and the assumptions of Section 3.1. Compare the elasticity of marginal costs with respect to output, Γ_Λ, in the two supply regimes (superscripts P and Q) for the following two cases of ownership of the E-good.*

1. *If the* essential good (the E-good) is owned domestically *($\iota = 1$), then*

$$\Gamma_\Lambda^Q = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha} \quad > \quad \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi} = \Gamma_\Lambda^P \quad > \quad 0. \tag{11}$$

2. *If the* essential good (the E-good) is owned abroad *($\iota = 0$), then*

$$\Gamma_\Lambda^Q = \frac{\sigma + \varphi - \frac{\alpha}{\theta}(\sigma-1)}{1+\alpha(\sigma-1)}, \qquad \textit{and} \qquad \Gamma_\Lambda^P = \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta(\sigma+\varphi)} > 0. \tag{12}$$

   In this case,

$$\Gamma_\Lambda^Q \underset{(<)}{>} \Gamma_\Lambda^P \qquad \textit{if and only if} \qquad \sigma + \varphi \underset{(<)}{>} \frac{\sigma-1}{\theta}. \tag{13}$$

Proof. The proof is provided in Appendix D.2. □

Supply constraints make the constrained factor's price more responsive to demand; compare $\Gamma^P_{p_E}$ and $\Gamma^Q_{p_E}$ in (8) and (9). All else being equal, this steepens the Phillips curve, meaning $\Gamma_\Lambda^Q > \Gamma_\Lambda^P$. Indeed, when the constrained factor is owned domestically, this is the supply regime's only effect on the slope of the Phillips curve (Case 1 above). In contrast, with foreign ownership (Case 2), there also is a wealth effect on labor supply. This can be seen from the terms in (11) and (12). Absent the wealth effect (for $\sigma = 0$), the terms Γ_Λ^Q and Γ_Λ^P in Case 1 and 2 are identical – they do not depend on ownership. For $\sigma > 0$, this is no longer the case. Higher output means a higher price for the constrained factor. If the proceeds accrue to Foreign, a higher price means that the domestic economy becomes less productive in converting labor into consumption, dampening the elasticity of the wage to output. The conditions in (13) show that with foreign ownership and if $\sigma > 1$, the Phillips curve may even flatten because of supply constraints, and the slope may potentially turn negative. This can arise when θ is small (such that the price of the constrained factor rises steeply with output) and labor supply is fairly elastic (φ is small) but households are unwilling to intertemporally substitute consumption (σ is large).[13]

Summing up, supply constraints raise $\widetilde{\kappa}$ (the slope of the Phillips curve) if either the constrained factor is owned domestically or the constrained factor is held abroad and the wealth effects on labor supply are weak. Then, from the supply side, supply constraints make any monetary stimulus more inflationary – see (6) – and, from the supply side, they *reduce* the risk of indeterminacy by Proposition 1.

3.5 Supply constraints and the slope of the IS curve

Supply constraints, however, also have a differential effect on different types of income. Emphasizing this is an important contribution of the current paper. Supply constraints directly affect the cyclicality of revenue from the constrained factor. And, indirectly, they affect the cyclicality of labor income through input substitution. In the setting here, this matters for aggregate demand (and the slope of the IS curve) because different agents can have different sources of income and have different marginal propensities to consume. Appendix C.2.12 shows that the slope of the IS curve in (5) relates to the elasticity of savers' consumption to aggregate output, Γ_{C_S}, through the following relationship:

$$\frac{1}{\widetilde{\sigma}} = \frac{1}{\sigma \, \Gamma_{C_S}}. \tag{14}$$

In the current setting, the supply of bonds in equilibrium is zero,[14] so all agents in equilibrium consume exactly their own income. The intuition then is familiar from, for example, Bilbiie (2008). The only agents whose demand is directly interest elastic are the saver households. A higher real interest rate raises savers' demand for bonds. For the bond market to clear, savers' incomes in equilibrium then have to fall to reduce the demand for bonds. The more procyclical the savers' income is (the more positive Γ_{C_S} is),

[13]While we document the effect of supply constraints for an arbitrary elasticity of substitution (θ), Blanchard and Riggi (2013) discuss the role of the wealth effect for long-run labor supply only and with a Cobb Douglas production function ($\theta = 1$) and only for what we call the fixed-price regime.

[14]This is by virtue of the assumption of balanced trade, which is needed for analytical tractability. Both hand-to-mouth households and Foreign, thus, have an MPC of 1. The numerical simulations in Section 4 will use a Foreign MPC that is less than 1, so that trade no longer is balanced.

the less of a fall in aggregate income is needed to bring this about. If, however, savers' income is countercyclical ($\Gamma_{C_S} < 0$), in equilibrium a higher real interest rate must go hand in hand with *higher* output. The next few lines highlight how the cyclicality of savers' income, in turn, is related to the incomes of the economy's other agents – namely, the hand-to-mouth households and the Foreign economy. Thereafter, we discuss how the exposure to different income streams shapes the demand-side effects of supply constraints. In the economy at hand, the domestic distribution of income is governed by

$$\Gamma_C = (1-\lambda)\,\Gamma_{C_S} + \lambda\,\Gamma_{C_H}. \tag{15}$$

Domestic income (C is aggregate consumption, which here is equal to GDP) is shared between savers (mass $1-\lambda$) and hand-to-mouth households (mass λ). The international distribution of income, in turn, is given by the following:

$$1 = [1-\alpha(1-\iota)]\,\Gamma_C + \alpha\,(1-\iota)\,\left[\Gamma_{p_E} + \Gamma_E\right]. \tag{16}$$

Foreign's source of income is the E-good. Thus, $\alpha(1-\iota)$ is the steady-state share of Home's output that goes to Foreign. How a marginal increase in output ("1" on the left-hand side) is allocated between domestic income, Γ_C, and foreign income (the second term on the right-hand side) directly relates to how cyclical the revenues from the E-good are, term $[\Gamma_{p_E} + \Gamma_E]$; this cyclicality of the revenues in turn depends on the supply regime – recall the expression for Γ_{p_E} and Γ_E in (8) and (9).

Focus on (15) first. In Proposition 3, we look at the case of domestic ownership of the E-good ($\iota = 1$) – where, by (16), $\Gamma_C = 1$ – and spell out how ownership of the different income sources *across* domestic households but *within* the same country shapes the effect of supply constraints on aggregate demand (the slope of the IS curve).

Proposition 3. *Consider the model in Section 2 and the assumptions listed in Section 3.1. In addition, suppose that all of the essential good (the E-good) is owned domestically ($\iota = 1$). Let $\Gamma_{C_S}^{Q,(i)}$ and $\Gamma_{C_S}^{P,(i)}$ mark the elasticity of savers' income with respect to output in cases $i = 1, 2, 3$ below in the two supply regimes for the E-good (superscripts P and Q).*

1. *$\nu = \vartheta = 0$: hand-to-mouth households do not receive profits or revenues from the*

essential good. Then,

$$\Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)} \quad < \quad 0.$$

2. $\nu = 0$, $\vartheta > 0$*: hand-to-mouth households do not receive profits, but a share of revenues from the essential good. Then,*

$$\Gamma_{C_S}^{Q,(2)} - \Gamma_{C_S}^{P,(2)} \quad < \quad \Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)}.$$

3. $\nu = \vartheta > 0$*: hand-to-mouth households receive the same positive share of profits as of revenue from the essential good. Then,*

$$\Gamma_{C_S}^{Q,(3)} - \Gamma_{C_S}^{P,(3)} \quad > \quad \Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)}.$$

Proof. The proof is provided in Appendix D.3. □

All else being equal, higher output means higher demand for the essential good and for labor. Labor income and income from the E-good rise disproportionately, particularly if the supply of one of the factors of production is constrained; they rise directly or because of substitution. Profit income thereby falls when output rises and even more so when there are supply constraints. Case 1 shows that if savers own all non-labor sources of income (and hand-to-mouth households none, $\nu = \vartheta = 0$), supply constraints unambiguously render savers' income less procyclical – the IS curve steepens, that is, demand becomes more interest sensitive. Indeed, supply constraints might turn Γ_{C_S} negative. This effect is even stronger (Case 2) if hand-to-mouth households share in the revenue from the essential good (a procyclical source of income). On the other hand, the effect is weaker if hand-to-mouth households not only share in the revenue from the essential good but also in profit income (Case 3).

Next, Proposition 4 focuses on the effect of supply constraints on the distribution of income *across* countries, recall (16). For ease of exposition, the E-good here is entirely owned by Foreign so that none of the income from the E-good accrues to households in Home ($\iota = 0$). The value of ϑ then is irrelevant, and we look at different cases for the ownership of domestic profits only.[15]

[15]Note that the cases specified in Proposition 3 and 4 are thus not identical.

Proposition 4. *Consider the model in Section 2 and the assumptions listed in Section 3.1. In addition, suppose the essential good (the E-good) is fully owned by Foreign ($\iota = 0$). The superscripts below refer to the two supply regimes for the E-good (superscripts P and Q) and the following three cases:*

1. $\nu = \lambda$: *representative household in Home.*

2. $\nu = 0$: *hand-to-mouth households do not receive profits.*

3. $\nu > 0$: *hand-to-mouth households receive a positive share of profits.*

With these definitions of the cases, the following are true:

$$\Gamma^{P,(1)}_{C_S} > 0 \qquad \text{and} \qquad \Gamma^{Q,(1)}_{C_S} < \Gamma^{P,(1)}_{C_S}, \tag{17}$$

and

$$\Gamma^{Q,(3)}_{C_S} - \Gamma^{P,(3)}_{C_S} \underset{(<)}{>} \Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S} \quad \text{if and only if} \quad \sigma + \varphi \underset{(<)}{>} \frac{\sigma - 1}{\theta}. \tag{18}$$

Proof. The proof is provided in Appendix D.4. □

Equation (17) focuses on the case where there is heterogeneity across borders but not domestically. This case is important because one can show that in the fixed-price scenario the slope of the IS curve is strictly positive, $\Gamma^{P,(1)}_{C_S} > 0$. Supply constraints, however, reduce the procyclicality of savers' income – $\Gamma^{Q,(1)}_{C_S} < \Gamma^{P,(1)}_{C_S}$ – thus, steepening the IS curve; in fact, the constraints may cause the slope of the IS curve to invert. The inequalities in (18), in turn, show that if the revenue from the essential good accrues to Foreign, the wealth effect on labor supply also matters for the slope of the IS curve, provided there is heterogeneity in Home. The reason is simple. A strong wealth effect can reverse the countercyclicality of profits, with a corresponding effect on the cyclicality of the income of those who claim the profits.

Summing up, we have derived conditions under which supply constraints tend to steepen the IS curve (making aggregate demand more interest sensitive) since they reduce the procyclicality of savers' incomes.[16] These conditions are savers' ownership of the income from profits and other agents' ownership of the income from the good subject to supply constraints. The degree of cyclicality of the income streams and of marginal costs depends

[16]The same conditions, thus, flatten the aggregate demand relation in inflation-output space.

in turn on the model's deep parameters: the elasticity of substitution in production, θ, intertemporal substitution, $1/\sigma$, the Frisch elasticity $1/\varphi$, the weight of the E-good in production, α, and nominal rigidities, ψ.[17] It is important to keep in mind that what matters for the above mechanisms is after-tax income. This means that fiscal policy, too, would shape the results.

Next, we turn to a quantitative assessment of the full model. There, energy is used in both consumption and production. Furthermore, we allow for wage rigidity, a marginal propensity to consume of Foreign that is less than unity, and for energy-related subsidies.

4 Numerical analysis and policy implications

This section calibrates the model to the German economy and associates the E-good with energy. We analyze the working of the model through three scenarios. In the "fixed price" scenario, the baseline, energy is supplied perfectly elastically at a given price. In the "fixed supply" scenario, instead, the supply of energy is fixed. All other features remain unchanged; these two scenarios reflect rather "normal" times. Finally, we design what we call the "crisis" scenario. This is aimed at capturing further aspects important for the German energy crisis that began in the run-up to Russia's 2022 invasion of Ukraine.[18]

4.1 Parameterization

Table 1 provides the parameterization for all three scenarios. We start with the baseline calibration – the "fixed price" scenario. One period in the model is taken to be a quarter. Turning first to preferences, the discount factor β implies a two percent annualized real rate of interest in the steady state. We set σ equal to 2. This value implies an intertemporal elasticity of substitution of 1/2, which is at the upper end of the range reported in Havránek (2015). We calibrate φ to 3, giving a Frisch elasticity of 1/3, which is in

[17] Appendix E shows directly how these parameters affect the validity of the Taylor principle in a special case where $\iota = 0$ and $\nu = \lambda$. As suggested by result (17) of Proposition 4, in this case indeterminacy can arise even when the central bank adheres to the Taylor principle – but only if there are supply constraints.

[18] All the impulse responses that we show will be based on a linearization of the model.

Table 1 Parameterization

Preferences		Production			Energy, Foreign			Government	
β	0.995	ε	11		ι	0.333		τ^y	0.1
σ	2	ψ	507		p_E	0.101		τ^w	0.1
φ	3	α	0.077		$\mu_{F,1}$	0.25		ν	0
χ	0.778	θ	0.1		$\mu_{F,2}$	0.02		ϑ	0
λ	0.24							$\overline{T}_H$	0.012
$\bar{e}$	0.125	Labor market						ϕ_Π	1.5
γ	0.239	ε^w	11					τ_E^c	0
η	0.1	ψ^w	507					τ_E^f	0
Change for fixed-supply scenario					Additional changes for crisis scenario				
p_E	flexible	$\xi_{E,t}$	1.5	const.	$\xi_{E,t}$	1.379	const.	τ_E^c	0.33
					ψ^w	0		τ_E^f	0.33

Notes: "Fixed-price" calibration in top panel. Changes relative to baseline for scenario "fixed-supply", and relative to that for scenario "crisis" in bottom panel. See main text for details. "Flexible" indicates that in the fixed-supply and crisis scenarios p_E is determined in equilibrium, while it is constant in the "fixed-price" scenario. "const." indicates that ξ_E is constant in the "fixed supply" and "crisis" scenarios, while it is determined in equilibrium in the "fixed price" scenario.

the middle of the range reported in Elminejad et al. (2023).[19] The disutility of work χ is set to normalize the steady-state labor supply of households to 1. The share of hand-to-mouth households is $\lambda = 0.24$, which follows estimates for Germany in Slacalek, Tristani and Violante (2020). $\bar{e}$ is set so that subsistence energy consumption is 25 percent of steady-state energy consumption, following Fried, Novan and Peterman (2022). We set γ so that, in the baseline steady state, households spend 4 percent of GDP on energy.[20] The elasticity of substitution in consumption η is set to 0.1, which is within the range reported in Bachmann et al. (2022).

Turning to production next, the own-price elasticity of demand is $\varepsilon = 11$, a conventional value. The price adjustment costs ψ match a slope of the pencil-and-paper Phillips curve of 0.1. In a Calvo setting this would map onto prices for *non-energy goods* being adjusted, on average, once a year. We set the elasticity of substitution between different types of labor (ε^w) and wage adjustment costs (ψ^w) to the same values as for prices. Energy's weight in production (α) is set to obtain costs of energy in production of 8 percent of

[19] Using the expression derived in Swanson (2012) the implied risk aversion is $(1/\sigma + 1/\varphi)^{-1} = 1.2$.

[20] The parameterization here considers sources of energy with local markets (natural gas, coal, and electricity, but not oil). We calibrate the expenditure shares relative to GDP using 2021 German data on primary energy usage, see BDEW (2023). The relative share of households and firms follows from Eurostat's data on energy consumption by sector (product code: ten00124).

GDP. The elasticity of substitution in production (between energy and labor) is the same as in consumption – namely, $\theta = 0.1$ – and is in line with Bachmann et al. (2022).

Regarding the supply of energy, we assume that a share $\iota = 0.333$ of energy is owned by domestic households – in line with German import shares for primary energy. In the fixed price baseline, the price is set such that (along with the assumptions made about preferences and production) firms' energy usage takes on a value of $E = 1$ in the steady state. This gives $p_E = 0.101$. In the baseline, this price is fixed. As for the demand by Foreign, we set $\mu_{F,1}$ equal to 0.25. This means that for each additional euro in energy revenue, Foreign orders a fourth of a euro's worth of goods produced in Home.[21] Next, we set the debt elasticity of foreign demand to a value that is small but large enough to stabilize net foreign assets at the targeted long-run value of zero; $\mu_{F,2} = 0.02$.

Turning to the government, we assume that it sets τ^y and τ^w so that – in the steady state – there are no distortions associated with firms' or workers' market power. We do so primarily because the propositions in Section 3 do so as well. Further, we assume that hand-to-mouth households do not receive any profits in the economy ($\nu = 0$) nor any revenues associated with domestic energy ownership ($\vartheta = 0$). The government provides a transfer of $\overline{T}_H = 0.012$ to hand-to-mouth households that ensures that both household types have the same baseline steady-state income. We do this to make the results easy to understand. Turning to monetary policy, we set the response to core inflation to $\phi_\Pi = 1.5$. The government does not implement energy price subsidies.

Fixed-supply scenario. This scenario shares all parameter and steady-state values with the fixed-price baseline. However, it keeps the supply of energy fixed over the cycle.

The crisis scenario. Section 3 provided conditions under which the Taylor principle can be invalidated. To obtain those results, the following were important elements: (1) a factor of production owned externally receiving notable weight in production, (2) the

[21] Drechsel and Tenreyro (2018) discuss the effects of commodity price booms on commodity exporters and find a sizable increase in imports after an increase in commodity prices, speaking in favor of high MPCs out of energy revenues. Arroyo Marioli and Vegh (2023) document a large procyclicality of fiscal policy for commodity exporters. Johnson, Rachel and Wolfram (2023) consider a unit MPC for an energy exporter, alluding to borrowing constraints and risks associated with accumulating financial assets (the risk of sanctions, say). In light of this, we consider our parametrization as conservative.

factor being in scarce supply so that prices are responsive, and (3) income streams supporting aggregate demand. The energy crisis that afflicted Germany in the run-up to and after the Russian invasion of Ukraine arguably provided some of these elements:

In recent years Germany imported roughly two thirds of its primary energy. Before the war, Russia alone provided one-third of German primary energy, natural gas in particular. Panel (a) of Figure 1 shows wholesale prices of natural gas in Europe, Japan, and the

Figure 1 Natural gas prices and trade flows of Germany

Notes: Panel (a) plots monthly natural gas prices in US dollars per mmbtu. Europe: Netherlands Title Transfer Facility; US: spot price at Henry Hub, Louisiana. Japan: Liquefied natural gas import price. Source: World Bank Commodity Price Data. Panels (b) and (c) plot quarterly nominal trade flows of Germany in billions of euros with the rest of the world and Russia, respectively. Source: International Monetary Fund Direction of Trade Statistics. The vertical dashed line marks the end of 2022Q1.

US; all denoted in US$. The vertical dashed line marks the end of 2022Q1 (the Russian invasion of Ukraine began on February 24, 2022). European prices show a run-up starting in 2021. To date, they still are about twice as high as before 2021. What is more, the fluctuations in prices after 2021 are unique to the European market. They suggest (or are at least consistent with) conditions of inelastic supply. This is perfectly in line with the frantic rerouting of supply and trade that followed the Russian invasion. Panels (b) and (c) plot the value of German trade – imports and exports alike. Panel (b) shows Germany's exports to and imports from the rest of the world in nominal terms. Both rose from 2021 onward, peaking in mid-2022 before falling in lockstep. Panel (c) focuses on the value of direct trade with Russia, which rose sharply before the war on the back of higher energy prices. Thereafter, with sanctions in place, trade with Russia fell precipitously.[22]

[22]In spite of the shortages, the remaining three German nuclear power plants were decommissioned only 3.5 months later than was originally planned, namely, in mid-April 2023. The supply of nuclear fuel, too, came historically from Russia.

The crisis scenario thus entertains a regime of fixed energy supply and adds to this a permanent cut of supply of 8 percent (calibrated so that energy prices are twice as high as in the baseline). This relates to conditions (1) and (2) mentioned above.

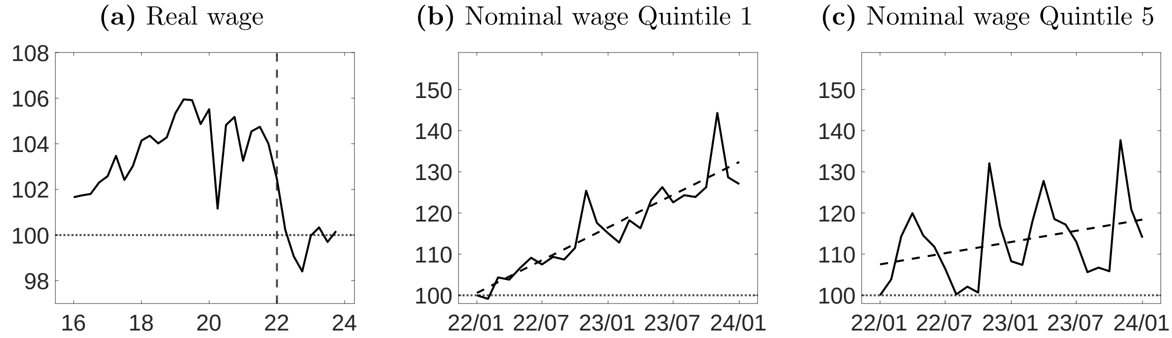
The final element, condition (3) concerns how – once the supply constraint binds – the costs and incomes of agents with high marginal propensities to consume move with a still higher energy price. On the negative side, for hand-to-mouth households the expenditures for energy rise. On the positive side, this may be cushioned by fiscal transfers and labor income may rise if the rise in energy prices is due to demand. Regarding fiscal transfers, the number of programs that the German government implemented to "protect" households and firms from higher energy costs is legion. The crisis scenario assumes that fiscal policy lets two thirds of price changes pass through to households or firms, "protecting" households and firms from the remainder – that is, it sets $\tau_E^c = \tau_E^f = 0.33$. The scenario is conservative in that it only refers to prices in deviation from the new higher-price steady state.[23] Lastly, and perhaps most contentiously, the crisis scenario assumes that nominal wage rigidities no longer restrain wage demand so that $\psi^w = 0$. The scenario is built around a tight labor market in Germany, in which the unemployment rate for prime-aged workers stood at 3 percent at the end of 2021 – falling further to 2.8 percent by 2023Q1.[24] In addition, in this environment, wage flexibility may have been actively encouraged politically.[25] Wage developments are shown in Figure 2. On the back of higher energy costs, real wages fell sharply from mid-2021 to mid-2022, but then rapidly stabilized (Panel (a)) while inflation remained elevated. The recovery of real wages came with strong increases in nominal wages. Nominal wages rose about ten percent more for the lowest earnings quintile (Panel (b)) than for the top quintile (Panel (c)). The crisis scenario we build entertains the reading that this may be indicative of both wage

[23] The International Energy Agency's policy database lists energy price related policy measures. Among several, they include: double-digit cuts to taxes on natural gas and to electricity surcharges; postponing CO2-surchanges; energy cost transfers to poorer households, students, pensioners; and non-targeted transfers to all ("Energiepreispauschale"), besides transfers to energy-intensive firms, culminating in explicit price caps on electricity/natural gas/heating costs for households and firms ("Energiepreisbremsen").

[24] ILO definition. Similar patterns emerge for employment rates or participation, or other age groups.

[25] For example, since October 2022, employers in Germany have been able to grant their employees an amount of up to 3,000 euros free of tax and contributions as a voluntary benefit over and above the regular wage–the so-called "inflation compensation bonus", which the federal government has introduced by law. Also, the minimum wage was increased repeatedly.

Figure 2 Tight labor market in Germany



Notes: Panel (a) plots the seasonally adjusted quarterly real wage index. The vertical dashed line marks the end of 2022Q1. Panels (b) and (c) show the monthly nominal wage index for the lowest and highest quintiles of the wage distribution. The series we use start in 2022. Therefore, we did not seasonally adjust the series. This explains the seasonal spikes in Panels (b) and (c) (summer gratification and Christmas gratification). Source: German Federal Statistical Office.

flexibility and of wage pressures that support demand.

4.2 Steady states

Table 2 reports the steady state of the economy. In each block, the left column refers to the baseline fixed-price scenario and the fixed-supply scenario which share the same steady state. The right column shows the "crisis" scenario's steady state. In the crisis scenario, the reduction in energy supply and the ensuing energy price increase mean that the expenditure share for energy in GDP doubles from 12 to 24 percent. Firms and households reduce their use of energy. Hand-to-mouth households reduce consumption of goods and energy by more than savers. This is the case even though the hand-to-mouth households raise their labor supply more. Consumption inequality increases, and GDP falls. All of this is in line with a sharp fall of the real wage.

4.3 Transmission of energy shocks in normal times

In order to allow a comparison with the literature, Figure 3 shows the transmission of an *exogenous* increase in the energy price in the fixed-price scenario (solid lines) and the transmission of a cut to energy supply with fixed supply (dashed-dotted lines). The shocks are calibrated to deliver a twenty percent increase in the energy price and have a persistence of 0.97 – following Blanchard and Galí (2009). Because the two shocks

Table 2 Steady states in the scenarios

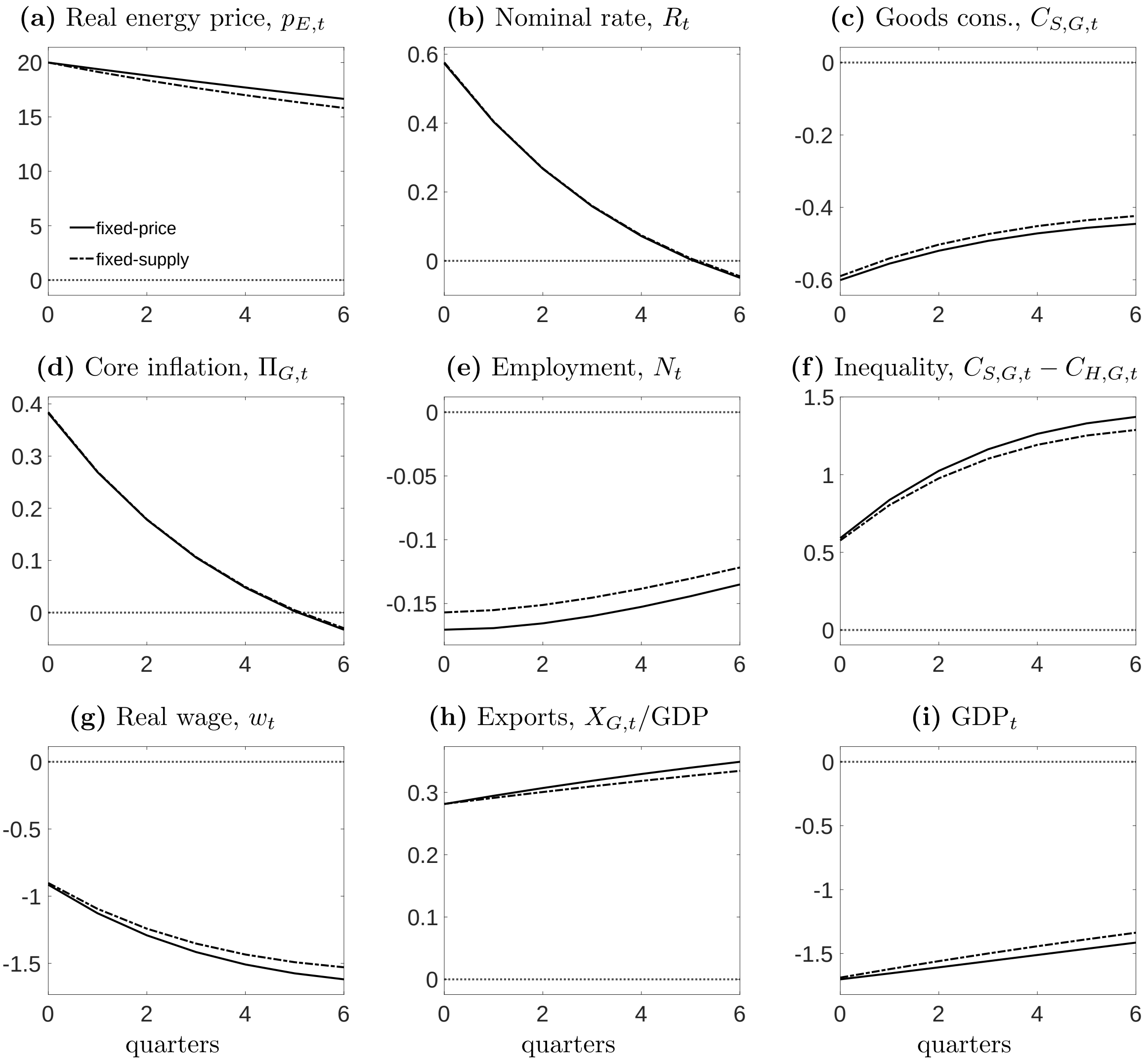
	Scenario				Scenario		
	P/Q	crisis			P/Q	crisis	
Consumption				Production, firms			
$C_{H,E}$	0.5	0.44	Energy cons. spender	GDP	1.259	1.176	
$C_{S,E}$	0.5	0.45	Energy cons. saver	Y_G	1	1.003	Production
C_E	0.5	0.448	Total energy cons.	p_G	1.309	1.269	Real goods price
$C_{H,G}$	0.923	0.835	Goods cons. spender	Λ	1.309	1.269	Real marginal costs
$C_{S,G}$	0.923	0.861	Goods cons. saver	D	0.131	0.127	Real profits
C_G	0.923	0.855	Total goods cons.				
Labor market				Energy, Foreign			
N_H	1	1.028	Labor spender	ξ_E	1.5	1.379	Energy supply
N_S	1	1.007	Labor savers	E	1	0.932	Energy used in prod.
N	1	1.012	Aggregate labor	p_E	0.101	0.205	Real energy price
w	1.209	1.069	Real wage	X_G	0.077	0.148	Real exports
Government				Implied ratios, in percent			
T_H	-0.071	-0.06	Lump-sum spender	$\frac{p_E C_E}{\text{GDP}}$	4	7.788	energy cons./GDP
T_S	-0.071	0	Lump-sum saver	$\frac{p_E E}{\text{GDP}}$	8	16.212	energy cost/GDP
R	1.005	1.005	Gross nominal rate	$\frac{p_E \xi_E}{\text{GDP}}$	12	24	total energy exp./GDP

Notes: The table reports steady-state values for all variables in the respective scenarios. P/Q refers to the fixed-price and fixed-supply scenarios. Note that T_H and T_S include taxes, profit income, and revenue from domestic energy ownership. GDP is as defined in Appendix B.4. Furthermore, in each steady state, $p_E^c = p_E^f = p_E$, $\Pi = \Pi_G = \Pi_W = 1$, and $b = 0$. In addition, steady-state values for C_H and C_G could be derived from equation (1), the value of Y^* from the definition of foreign income in Section 2.6.

are calibrated to have the same price effect, they also come with similar implications for quantities. A twenty percent increase in energy prices (Panel (a)) is associated with a decline in GDP of about 1.5 percent (Panel (i)). This is broadly in line with empirical estimates.[26] Not only does GDP fall, but so do wages (Panel (g)) and employment (Panel (e)). The distributional impact of the shock is pronounced. Savers' consumption (Panel (c)) falls but hand-to-mouth households' falls more than twice as much. To show this immediately, Panel (f) plots a measure of consumption inequality, namely, the difference between the consumption response of savers and of hand-to-mouth households. The reason is that hand-to-mouth households exclusively rely on falling labor income whereas savers' incomes are partially cushioned by rising markups and energy revenues. Inflation (Panel (d)), meanwhile, rises. Clearly, such fundamental energy shocks are supply shocks. We,

[26] See, for example, the effect of inventory demand shocks on global activity in Baumeister and Hamilton (2019), the SVAR-based findings in Blanchard and Galí (2009) and Blanchard and Riggi (2013), and the oil supply news shocks identified by Känzig (2021).

Figure 3 Fundamental energy shock under elastic and inelastic energy supply



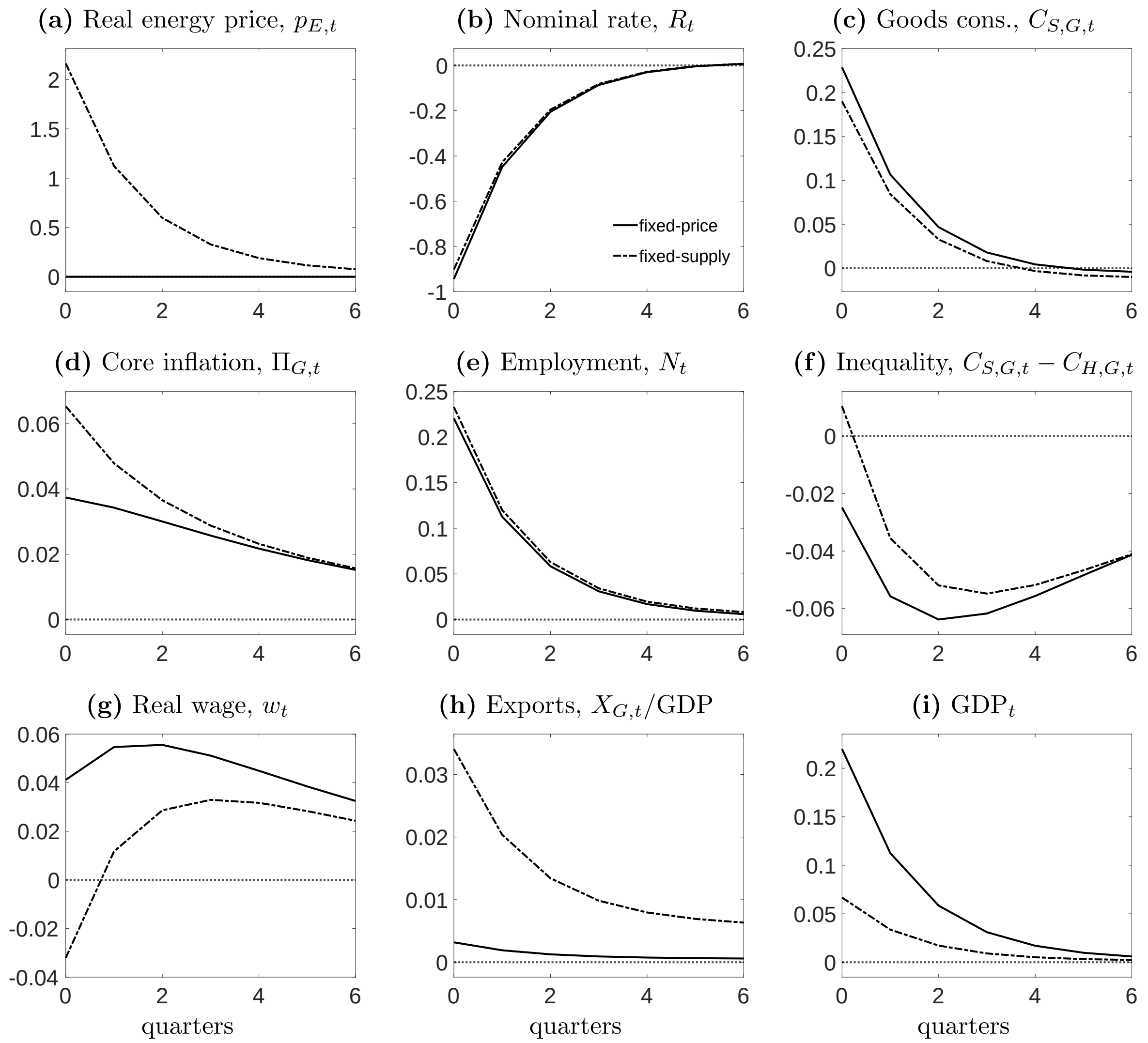
Notes: The figure plots impulse responses to an energy price shock in the fixed-price regime and an energy supply shock in the fixed-supply regime; scaled to raise the real energy price by twenty percent on impact and with persistence of 0.97. All responses give percentage deviations from the steady state. The exception is the response of exports (in percent of steady-state GDP). Interest rates and inflation rates are in annualized percentage points. Inequality is the difference between the responses of consumption of savers and hand-to-mouth households.

instead, wish to analyze how supply constraints, once they are in place, shape the business cycle and the transmission of monetary and fiscal policy, in particular. We do this next.

4.4 Monetary and fiscal transmission and the supply regime

The current section explores how the supply constraints change the economy's response to monetary and fiscal policy. As discussed in Section 3, the constraints affect the transmission of policy both through the supply side and the demand side, affecting the slope of both the Phillips curve and the IS curve. Figure 4 plots the dynamics in response to

Figure 4 Monetary policy shock under elastic and inelastic energy supply



Notes: The figure plots impulse responses to a 25 basis point (not annualized) monetary easing in the fixed-price and the fixed-supply regime with persistence of 0.5. For a definition of the variables see the notes to Figure 3.

a monetary easing. The solid line in each panel represents the baseline where the energy price is fixed and the energy supply potentially abundant. The dashed line instead represents the fixed-supply situation where energy supply is constrained and the energy price is assumed to be responsive to economic activity.[27]

In either supply regime, the monetary easing (Panel (b)) induces savers to increase demand for goods (Panel (c)). Production rises on the back of rising aggregate demand, and so does employment (Panel (e)). Wage rigidity means that real wages move little (Panel (g)).

[27]Recall that the steady state level and steady state price of energy are the same in both scenarios.

While this rigidity alone dampens the distributional impact, hand-to-mouth households still tend to be more positively affected by the easing than savers: consumption of hand-to-mouth households rises by more than the consumption of savers (the entries in Panel (f) are negative). The reason is that savers lose from falling markups.

When energy supply is fixed (dashed lines), instead, the cyclical distribution of income changes markedly, with reverberations on the demand side. Now the energy market clears through a rise in the energy *price* (Panel (a)). For consumers and firms this is akin to a tax on the use of energy, a tax to the benefit of the owners of energy (savers and the foreign economy). As a result, the distributional consequences of the easing are less favorable for hand-to-mouth households (Panel (f)) and the rise in aggregate consumption is weaker. GDP (Panel (i)) rises by notably less, too, since a rising real value of imports more than compensates for the rise in exports (recall that Foreign's MPC is less than unity).

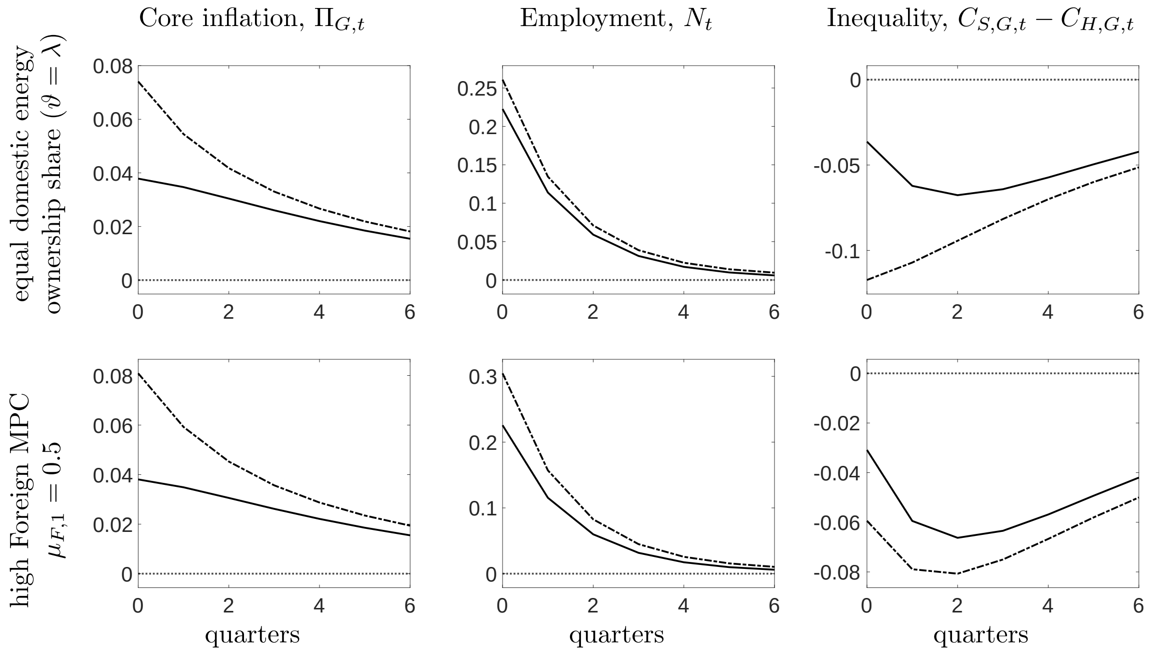
The Phillips curve, furthermore, clearly steepens (recall Section 3.4) upon a change toward a fixed-supply regime.[28] With energy prices responding to local activity, marginal costs become more responsive as well. For the same monetary easing, core inflation (Panel (d)) rises twice as much as in the regime with a fixed price of energy.

Taking the effects on the supply and demand side together, the fixed supply regime here means that a monetary easing is less expansionary and more inflationary. The impulse responses above are based on a linearized model. A monetary tightening then would have implications that are just of opposite sign. Framed in terms of the "sacrifice ratio" Figure 4, thus, shows that in order to bring down core (or headline) inflation by a certain amount, once energy supply constraints are in place monetary policy has to sacrifice notably less real activity and consumption.

Figure 5 zooms in further on the role of two of the demand-side determinants of the transmission of monetary shocks. The first row assumes that the domestic energy supply is owned not only by savers but in equal measure by hand-to-mouth households. This assumption, therefore, means that hand-to-mouth households no longer pay an implicit tax when energy prices rise. The second row shows the transmission of monetary policy

[28]For visual impression, Appendix F plots the Phillips curve's slope under the assumptions of Section 3.

Figure 5 Monetary shock: Supply regime and distribution of income

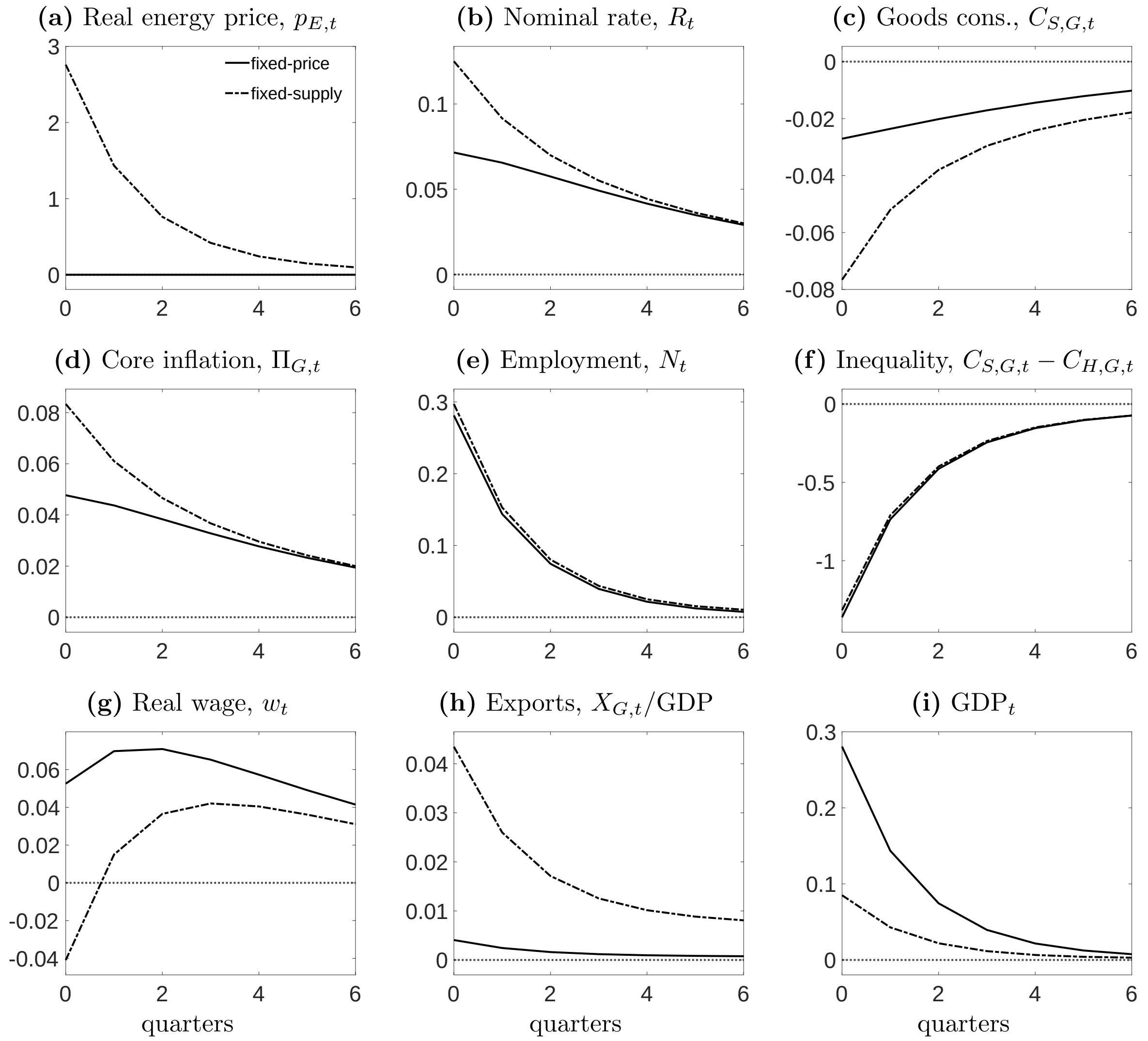


Notes: Same as Figure 4, but varying parts of the calibration.

if Foreign's MPC out of energy revenues is larger than in the baseline. A higher foreign MPC means that when energy prices rise foreign demand rises more strongly. Both of these alternative assumptions mean that supply constraints amplify the distributional and aggregate effects of monetary policy more than in the baseline parameterization. A monetary easing would be relatively more favorable for hand-to-mouth households.

The above discussion has dissected the implications of the supply constraint for a monetary policy shock. Figure 6 focuses on a temporary fiscal transfer to hand-to-mouth households, a common policy measure in recent years. The transfer is scaled to amount to one percent of hand-to-mouth households' steady state income. The fiscal transfer first and foremost directly benefits hand-to-mouth households. The transfer, thus, reduces "consumption inequality" between savers and hand-to-mouth households (Panel (f)) in either supply regime. However, with supply constraints, the same fiscal stimulus leads to higher core inflation (Panel (b)) at virtually the same employment response and at a substantially muted response of consumption (Panel (c)) and GDP (Panel (i)): the transfer multiplier is curtailed in this scenario. The reason for this is that with supply constraints redistributing from low- to high-MPC households increases demand and thereby the energy price (Panel

Figure 6 Transfer shock under elastic and inelastic energy supply



Notes: The figure plots impulse responses to a fiscal transfer of one percent of steady-state income of hand-to-mouth households from savers to hand-to-mouth households in the fixed-price and the fixed-supply regime with persistence of 0.5. For a definition of the variables see the notes to Figure 3.

(a)). This raises firms' marginal costs so that core inflation increases by more. At the same time, the higher energy price redistributes resources to the Foreign economy, which dampens the expansion of GDP.

Local supply constraints render the constrained factor's price endogenous to local economic activity. For the case of energy, the current section showed that such constraints, thereby, likely make a given monetary or fiscal stimulus more inflationary, less expansionary, and less favorable for constrained (hand-to-mouth) households.

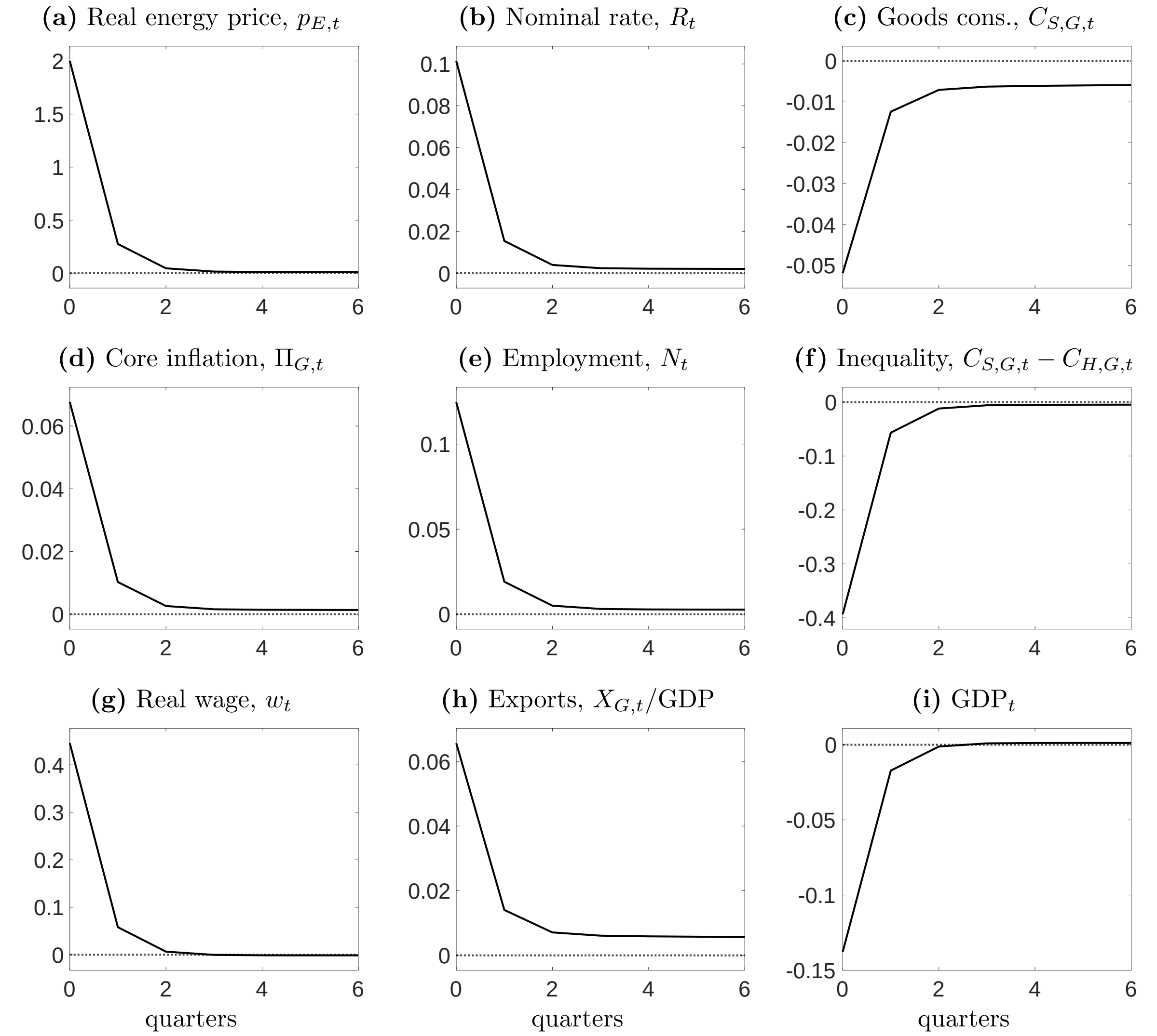
4.5 A feedback loop during a crisis?

Next, we turn to the "crisis scenario." This scenario renders the distributional dimensions of the supply constraint more important, for three reasons: first, the constrained factor has a larger weight in GDP to start with; second, wages are more flexible than usual so that domestic redistribution is more important and, third, fiscal policy shields households' and firms' budgets from movements in the constrained factor's price meaning that – at an individual level – these agents are not subject to the same transfer of income to the owners of energy when energy prices rise. The section underscores the theoretical result from Section 3 that supply constraints can give rise to self-fulfilling cyclical fluctuations. The mechanism for this is as follows. Consider a demand-driven boom for the sake of exposition. The crisis scenario has a fixed supply of energy. Aggregate demand in our environment comes from three types of agents: domestic hand-to-mouth households, the energy-exporting foreign economy, and domestic Ricardian households – ranked here from highest MPC out of income to lowest. The key to a demand-driven boom is that incomes are redistributed from savers to the former two types of agents. In the boom, wages rise to entice workers to supply labor, while energy prices (and, thus, Foreign's income) increase further amid high aggregate demand. As a result, markups fall. This leads to redistribution away from savers. Meanwhile, energy price related subsidies help sustain aggregate demand and the demand for energy in spite of high energy prices.

To see this mechanism at work graphically, we plot in Figure 7 impulse responses to a sunspot shock in the "crisis" scenario where numerically exactly one explosive root is missing to satisfy the Blanchard-Kahn conditions. Thus, there is exactly one degree of indeterminacy and room for exactly one possible sunspot shock. To compute the impulse responses to this shock, we use the methodology of Bianchi and Nicolò (2021). Theory determines the persistence of the sunspot shock, but not its magnitude – nor does it pin down the sign of the shock. The reader could, thus, just as well turn around the sign of the impulse responses and think of a recessionary sunspot shock.[29] In the figure, we somewhat arbitrarily anchor the shock's size such that it comes along with an increase in

[29] Recall that we look at a linear approximation of the model.

Figure 7 Sunspot shock in the "crisis" scenario



Notes: The figure plots impulse responses to a sunspot shock in the wake of which energy prices by two percent in the crisis scenario. For a definition of the variables see the notes to Figure 3.

the energy price of two percent (Panel (a)), with the magnitude of the price increase thus being a tenth of what we plot in Figure 3.[30]

With a 33 percent energy subsidy in place, energy prices for domestic users (households and firms) rise 1.3 percent. The sunspot supports higher core inflation for about three quarters (Panel (d)) reflecting the fact that firms face higher marginal costs. In line with the Taylor principle, the central bank raises the nominal interest rate more than one for one with core inflation (Panel (b)). A higher real interest rate means that savers'

[30]It is important to bear in mind that Figure 3 shows the effect of a fundamental shock to the energy price. Figure 7, instead, shows the response to a sunspot shock when the supply of energy is fixed. In Figure 7, the energy price rises *endogenously* with the higher price being a symptom of higher demand.

consumption falls (Panel (c)). Nevertheless, under the sunspot beliefs, output in Home rises about 0.1 percent (not shown). This increase requires a rise in employment (Panel (e)) even if the fact that households reduce their energy consumption (not shown) means that firms can increase their use of energy somewhat. On the demand side, the increase in economic activity is supported by two developments, each linked to the distribution of income. First, under the sunspot belief of higher energy prices, Foreign's revenues rise. In our calibration, Foreign uses one-fourth of the rise in revenues for buying goods from Home (Panel (h)). A second effect, which is directly linked to the heterogeneity of households, is that the domestic demand for consumption goods as a whole does not fall until sometime after the shock (not shown). The reason is the following: While savers retrench their consumption demand, the hand-to-mouth households' budgets initially are supported by a stronger labor market. Namely, labor demand rises – and so does the real wage (Panel (g)). Accordingly, the measure of "consumption inequality" decreases (Panel (f)). That said, in all these dynamics GDP falls because value added falls on the back of higher costs for energy imports (Panel (i)).

In the feedback loop, high energy prices are a *symptom* of high demand meeting supply constraints. Thus, the key policy implication is that monetary and fiscal policy can avoid the loop if they lean sufficiently strongly against demand (being tighter in a boom and looser in a recession). Appendix G provides a detailed discussion of the fiscal and monetary policy options that prevent the feedback loop. A key result is that a monetary response to headline, rather than core, inflation at conventional strengths would be sufficient to ensure determinacy in our crisis scenario, as would a response to input price inflation.

5 Conclusions

What is the effect of supply constraints on monetary transmission and macroeconomic stability? To provide an answer, we use a tractable New Keynesian open economy model with heterogeneous households in which the supply of an essential input factor is constrained. In this setting, the supply constraint changes the cyclicality of marginal costs and inflation. Moreover, through input substitution the supply constraint also affects the cyclicality of wages and profits and – of course – it directly affects the cyclicality of revenue from the constrained factor itself. We show how, depending on the ownership of these streams of income, supply constraints dampen or amplify the cyclicality of aggregate demand and the effectiveness of stabilization policy.

Next to theory, we provide a quantitative application to the German economy. Here we look at the supply constraints of energy that emerged at the beginning of 2022, around the time of the Russian invasion of Ukraine. In light of the ownership structure and use of energy in the German economy, energy supply constraints make both monetary and fiscal stimulus more inflationary and less effective at stimulating domestic demand.

We also entertain one, perhaps more subjective, interpretation of the economic environment in the energy crisis. Here, subsidies shield households and firms from energy price increases and a tight labor market gives rise to more flexible wages. We show that in this environment the conventional wisdom that the central bank should "see through" energy price movements could give rise to a sunspot equilibrium in which higher (lower) energy prices go hand in hand with higher (lower) economic activity. In sum, our analysis highlights that supply constraints make calibrating the fiscal and monetary response to the business cycle more difficult as the constraint shifts key macroeconomic elasticities.

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Online Appendix to:

Limited (Energy) Supply, Monetary Policy, and Sunspots

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— Not for publication —

A Details on the modeling in Section 2

This appendix provides further information on the paper's modeling of nominal wage and price rigidities.

A.1 Nominal wage rigidities

Our modeling of nominal wage rigidities follows Bilbiie, Känzig and Surico (2022) and Colciago (2011). There exists a range of differentiated labor services indexed by $s \in [0,1]$. Each type of household supplies all types of labor services to a "labor packer" that aggregates the different types of labor services into a homogeneous labor service N_t that serves as an input to production. Wage-setting decisions are made by labor-service-specific unions that maximize the utilitarian welfare of their members (the households). The members face quadratic adjustment costs for nominal wages. Given the wage that the union sets, workers supply all the hours that the labor packer demands.

Labor packer. The aggregate labor index is defined as the following

$$N_t = \left[\int_0^1 N_t(s)^{\frac{\varepsilon^w - 1}{\varepsilon^w}} ds \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}},$$

where $N_t(s)$ denotes differentiated labor services and $\varepsilon^w > 1$ measures the elasticity of substitution between labor services. Each labor service has the price $W_t(s)$. Given this wage, the resulting allocation problem yields the following labor demand curves

$$N_t(s) = \left(\frac{W_t(s)}{W_t} \right)^{-\varepsilon^w} N_t,$$

with the wage index $W_t = \left[\int_0^1 W_t(s)^{1-\varepsilon^w} ds \right]^{1/(1-\varepsilon^w)}$. Each household supplies all the types of labor services, and aggregate demand for labor type s is spread uniformly across the households. It follows that the individual quantity of hours worked in each service is common across households.

Wage setting. To describe the wage-setting process of each union, we are going to temporarily alter the preferences and budget sets of the households relative to the main text. However, as we will see, once we impose some of the implications of the union setting, we get back to those expressions; everything is consistent. Preferences for any $i \in \{H, S\}$ are now given by the following:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \chi \frac{\int_0^1 N_t(s)^{1+\varphi} ds}{1+\varphi} \right] \right\}.$$

Here, $N_t(s)$ denotes labor services supplied by the household to union s, where we impose the assumption that both types of households have to supply the same amount of each labor service and that each household supplies each type of labor service.
The modified budget constraint for spenders is given by the following:[31]

$$\begin{aligned} & P^c_{E,t} C_{H,E,t} + P_{G,t} C_{H,G,t} \\ = & \int_0^1 \left((1+\tau^w) W_t(s) N_t(s) - P_t \frac{\psi^w}{2} \left(\frac{W_t(s)}{W_{t-1}(s)} - 1 \right)^2 N_t \right) ds + P_t T_{H,t}. \end{aligned}$$

The right hand side now contains the labor income from all the types of labor services. In addition, households pay an adjustment cost when nominal wages of type s deviate from the wage last period, $\int_0^1 P_t \frac{\psi^w}{2} \left(\frac{W_t(s)}{W_{t-1}(s)} - 1 \right)^2 N_t dj$. $\psi_w > 0$ indexes these costs.
The saver's budget constraint, in turn, is given by the following:

$$\begin{aligned} & P^c_{E,t} C_{S,E,t} + P_{G,t} C_{S,G,t} + \frac{B_t}{1-\lambda} \\ = & \int_0^1 \left((1+\tau^w) W_t(s) N_t(s) - P_t \frac{\psi^w}{2} \left(\frac{W_t(s)}{W_{t-1}(s)} - 1 \right)^2 N_t \right) ds + P_t T_{S,t} + R_{t-1} \frac{B_{t-1}}{1-\lambda}. \end{aligned}$$

Before moving on to the union problem itself, we point out the following. First, we will look at a zero-inflation steady state later so that the resource cost of wage adjustment will be of second order. Given that we linearize to solve the model, we have dropped the costs from the main text for brevity. If, in addition, as will be the case in equilibrium, all unions demand the same amount of labor services, preferences and budget constraints will be back to the expressions in the main text. There is, as a result, no contradiction between the assumptions here and the description in the main text.
Each labor-service-specific union has a utilitarian objective. It sets its wage $W_t(s)$ so as to maximize the population-weighted utility of households in Home,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\lambda \frac{C_{H,t}^{1-\sigma}}{1-\sigma} + (1-\lambda) \frac{C_{S,t}^{1-\sigma}}{1-\sigma} - \chi \frac{\int_0^1 N_t(s)^{1+\varphi} ds}{1+\varphi} \right] \right\},$$

given the modified budget constraints described above as well as the definition of the consumption and price indexes. The first-order conditions yield the nonlinear wage Phillips curve in Section 2.2 after imposing the condition that the decision problem is symmetric across labor types s, meaning that all the unions will set the same wage in equilibrium.

[31]Remember also, that, for $i \in \{H, S\}$, $C_{i,t} = \left[\gamma^{\frac{1}{\eta}} \left(C_{i,E,t} - \bar{e} \right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{i,G,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$.

A.2 Production

Retailer. The representative competitive retailer transforms the differentiated inputs into the G-good according to the following production function

$$Y_{G,t} = \left[\int_0^1 y_{G,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the elasticity of substitution between the different differentiated inputs. The retailer takes prices $P_{G,t}(j)$ of intermediate inputs and $P_{G,t}$ of output as given. Profit maximization leads to the conventional Dixit Stiglitz demand function

$$y_{G,t}(j) = \left(\frac{P_{G,t}(j)}{P_{G,t}} \right)^{-\varepsilon} Y_{G,t},$$

with $P_{G,t} = \left[\int_0^1 P_{G,t}(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$ being the producer price index.

Differentiated goods. Each producer sets its price $P_{G,t}(j)$ so as to maximize the following:

$$\mathbb{E}_t \Bigg\{ \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{S,t+k}}{C_{S,t}} \right)^{-\sigma} \frac{1}{P_{t+k}} \Bigg[(1+\tau^y) P_{G,t+k}(j) y_{G,t+k}(j) - W_{t+k} N_{t+k}(j)$$
$$- P^f_{E,t+k} E_{t+k}(j) - \frac{\psi}{2} P_{G,t+k} Y_{G,t+k} \left(\frac{P_{G,t+k}(j)}{P_{G,t+k-1}(j)} - 1 \right)^2 \Bigg] \Bigg\},$$

where $\tau^y \geq 0$ is a constant sales subsidy and $\psi > 0$ indexes the quadratic price adjustment costs. $N_t(j)$ and $E_t(j)$ mark the labor input and energy input into production, respectively. $P^f_{E,t}$ indicates the energy price that *firms* pay (thus the subscript). Maximization is subject to the retailer's demand function and the production function (see Section 2.3). Next to the optimal factor input shares and the implied real marginal costs, the price setting problem implies the nonlinear price Phillips curve in Section 2.3. Equilibrium price adjustment costs are $\frac{\psi}{2} \frac{P_{G,t}}{P_t} Y_{G,t} \left(\Pi_{G,t} - 1 \right)^2$. As these are zero up to first order, we abstract from displaying them in profits in the main text.

B Model equations

This Appendix provides the model equations of the model of Section 2 all in one place. Appendix B.1 provides a complete list of variables and parameters. Appendix B.2 provides a complete list of model equations. Appendix B.3 on top of this defines the processes that govern the exogenous shocks. Appendix B.4 defines further variables that are discussed in Section 4 of the main text, but are not part of the core model shown in Section 2 of the main text.

B.1 List of variables and parameters

B.1.1 Variables

The model defines the following variables.

Prices, wages, and inflation rates:

(1) real price of E-good $p_{E,t}$

(2) real price of E-good that households pay $p^c_{E,t}$

(3) real price of E-good that firms pay $p^f_{E,t}$

(4) real price of G-good $p_{G,t}$

(5) real wage w_t

(6) nominal wage inflation $\Pi_{W,t}$

(7) marginal-CPI inflation Π_t

(8) producer price inflation $\Pi_{G,t}$

Consumption:

(9) hand-to-mouth household's consumption of E-good $C_{H,E,t}$

(10) saver's consumption of E-good $C_{S,E,t}$

(11) aggregate consumption E-good $C_{E,t}$

(12) hand-to-mouth household's G-good consumption $C_{H,G,t}$

(13) saver's G-good consumption $C_{S,G,t}$

(14) aggregate G-good consumption $C_{G,t}$

(15) hand-to-mouth household's consumption bundle $C_{H,t}$

(16) saver's consumption bundle $C_{S,t}$

(17) an aggregate consumption index C_t

Supply and production:

(18) supply of E-good $\xi_{E,t}$

(19) hand-to-mouth household's labor supply $N_{H,t}$

(20) saver's labor supply $N_{S,t}$

(21) aggregate labor supply N_t
(22) output of G-good $Y_{G,t}$
(23) use of E-good in production E_t
(24) profits D_t
(25) real marginal costs Λ_t

Fiscal and monetary policies:
(26) gross nominal interest rate R_t
(27) lump-sum transfer to hand-to-mouth household $T_{H,t}$
(28) lump-sum transfer to saver $T_{S,t}$

Foreign:
(29) real external savings b_t
(30) Foreign's revenue Y_t^*
(31) exports to Foreign $X_{G,t}$

At the core of the model, there are, thus, 31 variables. On top of this, there are the exogenous shocks used for the respective exercises. These shocks are discussed at the end in Appendix B.3.

B.1.2 Parameters

The model has the parameters listed below. We exclude steady-state values of model variables from the list for brevity.

Household and labor market:
(1) share of hand-to-mouth households λ
(2) time discount factor β
(3) risk aversion σ
(4) inverse Frisch elasticity φ
(5) scale disutility of hours worked χ
(6) elasticity of substitution between E-good and G-good in consumption η
(7) weight of E-good in consumption γ
(8) subsistence level of E-good in consumption $\bar{e}$
(9) wage subsidy τ^w
(10) Rotemberg cost of wage adjustment ψ^w
(11) elasticity of substitution between differentiated types of labor ϵ^w

Production and energy:
(12) elasticity of substitution between E-good and G-good in production θ

(13) weight of E-good in production α

(14) elasticity of substitution between differentiated goods ϵ

(15) Rotemberg cost of price adjustment G-good ψ

(16) sales subsidy τ^y

(17) supply of E-good if fixed ξ_E

(18) price of E-good if fixed p_E

Fiscal and monetary policy:

(19) share E-good supply owned by Home economy ι

(20) E-good subsidy consumption τ_E^c

(21) E-good subsidy production τ_E^f

(22) share of profits received by hand-to-mouth households ν

(23) share of Home's income from E-good received by hand-to-mouth households ϑ

(24) Taylor rule response to inflation ϕ_Π

(25) fixed transfer to hand-to-mouth households $\overline{T}_H$

Foreign economy:

(26) Foreign's MPC out of current income $\mu_{F,1}$

(27) Foreign's MPC out of wealth $\mu_{F,2}$

At the core of the model there are thus 27 parameters, excluding the steady-state values of model variables that appear in some equations. On top of this, there are the persistence parameters of exogenous shocks used for the respective exercises.

B.2 Model equations

In defining variables and showing the model equations, this section follows the flow of Section 2 in the main text.

B.2.1 Households in Home

The definition of the consumption-based price index, P_t gives the following relation:

$$1 = \left[\gamma {p_{E,t}^c}^{1-\eta} + (1-\gamma){p_{G,t}}^{1-\eta}\right]^{1/1-\eta}. \tag{B.1}$$

Budget constraint of hand-to-mouth households[32]

$$p_{E,t}^c C_{H,E,t} + p_{G,t} C_{H,G,t} = (1+\tau^w) w_t N_{H,t} + T_{H,t}. \qquad \text{(B.2)}$$

Consumption demand functions for E-good and G-good

$$C_{H,E,t} - \bar{e} = \gamma (p_{E,t}^c)^{-\eta} C_{H,t} \quad \text{(B.3)} \qquad C_{H,G,t} = (1-\gamma)(p_{G,t})^{-\eta} C_{H,t} \quad \text{(B.4)}$$

$$C_{S,E,t} - \bar{e} = \gamma (p_{E,t}^c)^{-\eta} C_{S,t} \quad \text{(B.5)} \qquad C_{S,G,t} = (1-\gamma)(p_{G,t})^{-\eta} C_{S,t} \quad \text{(B.6)}.$$

Euler equation of savers

$$C_{S,t}^{-\sigma} = \beta\, \mathbb{E}_t \{ C_{S,t+1}^{-\sigma} R_t / \Pi_{t+1} \}. \qquad \text{(B.7)}$$

This leaves us with 7 equations for 31 variables.

B.2.2 Labor market in Home

By the definition of the real wage

$$\Pi_{W,t} = w_t / w_{t-1}\, \Pi_t. \qquad \text{(B.8)}$$

Otherwise, we differentiate two scenarios.
If wages are flexible, we have:

$$w_t = \chi N_{H,t}^{\varphi} C_{H,t}^{\sigma} \quad \text{(B.9.a)} \qquad w_t = \chi N_{S,t}^{\varphi} C_{S,t}^{\sigma}. \quad \text{(B.10.a)}$$

If wages are rigid, instead of (B.9.a) and (B.10.a), we use

$$\psi^w \Pi_{W,t}(\Pi_{W,t} - 1) = \varepsilon^w \left(\frac{\chi N_t^{\varphi}}{\lambda C_{H,t}^{-\sigma} + (1-\lambda) C_{S,t}^{-\sigma}} - (1+\tau^w) \frac{\varepsilon^w - 1}{\varepsilon^w} w_t \right) + \psi^w \beta\, \mathbb{E}_t \left\{ \frac{\lambda C_{H,t+1}^{-\sigma} + (1-\lambda) C_{S,t+1}^{-\sigma}}{\lambda C_{H,t}^{-\sigma} + (1-\lambda) C_{S,t}^{-\sigma}} \Pi_{W,t+1}(\Pi_{W,t+1} - 1) \frac{N_{t+1}}{N_t} \right\}, \qquad \text{(B.9.b)}$$

$$N_{H,t} = N_{S,t}. \qquad \text{(B.10.b)}$$

The following defines aggregate consumption index C_t:

$$C_t = \lambda C_{H,t} + (1-\lambda) C_{S,t}. \qquad \text{(B.11)}$$

[32]The budget constraint of a saver household

$$p_{E,t}^c C_{S,E,t} + p_{G,t} C_{S,G,t} + b_t/(1-\lambda) = (1+\tau^w) w_t N_{S,t} + T_{S,t} + R_{t-1}/\Pi_t\, b_{t-1}/(1-\lambda)$$

is redundant by Walras' law.

This leaves us with 11 equations for 31 variables.

B.2.3 Production in Home

Imposing symmetry, the production functions of retailers and differentiated goods producers result in the following expression:

$$Y_{G,t} = \left[\alpha E_t^{\frac{\theta-1}{\theta}} + (1-\alpha) N_t^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{B.12}$$

The price Phillips curve is given by the following:

$$\begin{aligned}\psi\Pi_{G,t}(\Pi_{G,t}-1) = \varepsilon\left(\frac{\Lambda_t}{p_{G,t}} - (1+\tau^y)\frac{(\varepsilon-1)}{\varepsilon}\right) \\ + \psi\beta\,\mathbb{E}_t\left\{\left(\frac{C_{S,t+1}}{C_{S,t}}\right)^{-\sigma}\Pi_{G,t+1}(\Pi_{G,t+1}-1)\,\frac{Y_{G,t+1}}{Y_{G,t}}\,\frac{p_{G,t+1}}{p_{G,t}}\right\}.\end{aligned} \tag{B.13}$$

Here $\Pi_{G,t}$ is defined through the following

$$\Pi_{G,t} = p_{G,t}/p_{G,t-1}\,\Pi_t. \tag{B.14}$$

Optimal factor demand relates to marginal costs as

$$w_t = \Lambda_t(1-\alpha)\left(Y_{G,t}/N_t\right)^{\frac{1}{\theta}} \quad \text{(B.15)} \quad \text{and} \quad p^f_{E,t} = \Lambda_t\alpha\left(Y_{G,t}/E_t\right)^{\frac{1}{\theta}}, \quad \text{(B.16)}$$

which combined give the expression for real marginal costs in the main text.
Profits are given by the following (once more abstracting from the adjustment costs):

$$D_t = (1+\tau^y)p_{G,t}Y_{G,t} - w_tN_t - p^f_{E,t}E_t. \tag{B.17}$$

This leaves us with 17 equations for 31 variables.

B.2.4 Fiscal policy in Home

The government budget constraint is given by the following

$$\begin{aligned}D_t + p_{E,t}\,\iota\,\xi_{E,t} = \tau^y p_{G,t}Y_{G,t} + \tau^w w_tN_t \\ + (p_{E,t} - p^c_{E,t})C_{E,t} + (p_{E,t} - p^f_{E,t})E_t + \lambda T_{H,t} + (1-\lambda)T_{S,t}.\end{aligned} \tag{B.18}$$

Price subsidies are paid according to the following

$$p^f_{E,t} = p_{E,t}\left(p_{E,t}/p_E\right)^{-\tau^f_E} \quad \text{(B.19)} \quad \text{and} \quad p^c_{E,t} = p_{E,t}\left(p_{E,t}/p_E\right)^{-\tau^c_E}. \quad \text{(B.20)}$$

Here, as elsewhere, expressions in the form of variables that do not carry a time subscript refer to the steady state.
Transfers to hand-to-mouth households are given by the following:

$$\lambda T_{H,t} = \overline{T}_H + \nu \times (D_t - \tau^y p_{G,t} Y_{G,t}) + \vartheta p_{E,t}\, \iota\, \xi_{E,t} - \lambda \tau^w w_t N_{H,t} + \zeta_t. \tag{B.21}$$

Transfers to savers are implied from the government budget constraint.
This leaves us with 21 equations for 31 variables.

B.2.5 Monetary policy in Home

The government sets monetary policy according to the Taylor rule

$$R_t = R \cdot (\Pi_{G,t}/\Pi_G)^{\phi_\Pi} \cdot \exp\{\upsilon_t\}. \tag{B.22}$$

B.2.6 International trade and Foreign demand

Foreign's budget constraint is given by the following:

$$p_{G,t} Y_t^* = p_{G,t} X_{G,t} - b_t + b_{t-1} R_{t-1}/\Pi_t. \tag{B.23}$$

Foreign's revenues are given by the following:

$$Y_t^* = (1 - \iota)\, \xi_{E,t}\, p_{E,t}/p_{G,t}. \tag{B.24}$$

Foreign demand (Home's exports) is given by the following:

$$X_{G,t}/X_G = (Y_t^*/Y^*)^{\mu_{F,1}} \times \exp\left(-\mu_{F,2}(b_{t-1}/Y^*)\right). \tag{B.25}$$

This leaves us with 25 equations for 31 variables.

B.2.7 Supply regime for the E-good

One of the following equations appears, depending on the supply regime.
In the *fixed-price* regime we assume

$$\log(p_{E,t}) = \log(p_E) + \nu_{p_E,t}, \tag{B.26.a}$$

for some fixed value of p_E.
In the *fixed-supply* regime we assume

$$\log(\xi_{E,t}) = \log(\xi_E) + \nu_{\xi_E,t}, \tag{B.26.b}$$

for some fixed value of ξ_E.

Above, $\nu_{p_E,t}$ and $\nu_{\xi_E,t}$ are shocks to the price or to the supply of the E-good, respectively. This leaves us with 26 equations for 31 variables.

B.2.8 Market clearing

The labor market clears if

$$N_t = \lambda N_{H,t} + (1-\lambda) N_{S,t}. \tag{B.27}$$

The market for the E-good clears if

$$\xi_{E,t} = C_{E,t} + E_t, \tag{B.28}$$

with aggregate consumption of the good defined as

$$C_{E,t} = \lambda C_{H,E,t} + (1-\lambda) C_{S,E,t}. \tag{B.29}$$

The market for domestic products clears if

$$Y_{G,t} = C_{G,t} + X_{G,t}, \tag{B.30}$$

with aggregate consumption defined as

$$C_{G,t} = \lambda C_{H,G,t} + (1-\lambda) C_{S,G,t}. \tag{B.31}$$

This means there are 31 equations for 31 variables.

B.3 Exogenous shocks

In the various scenarios of Section 4 we plot impulse responses of the economy to three fundamental shocks: shocks to the price of the E-good, $\nu_{p_E,t}$, shocks to the supply of the E-good, $\nu_{\xi_E,t}$, shocks to monetary policy, ν_t, and transfer shocks, ζ_t. All these shocks follow an AR(1) process with zero mean.

Next, we also plot the transmission of sunspot shocks that arise when there is determinacy. When modeling these and solving for their transmission, we follow Bianchi and Nicolò (2021). This involves defining a forecast error, FE_t for an arbitrary jump variable (here we use C_t) as

$$\text{FE}_t = \exp(\log C_t - \mathbb{E}_{t-1} \log C_t).$$

and defining the potential sunspot shock as

$$\log \omega_t = \rho_\omega \log \omega_{t-1} + \varepsilon_{\omega,t} + \log \mathrm{FE}_t,$$

where ρ_w takes an arbitrary value of $\rho_w = 1.5$ and $\epsilon_{\omega,t}$ is white noise.

B.4 Definition of further variables used

The simulations refer to further variables that are auxiliary definitions.
Real gross domestic product is given by the following:

$$\mathrm{GDP}_t = p_{E,t} C_{E,t} + p_{G,t} C_{G,t} + p_{G,t} X_{G,t} - p_{E,t}(1-\iota)\xi_{E,t}.$$

When the central bank responds to "the change in nominal marginal costs (to input price inflation, that is)" in Appendix G.1 of the main text, the measure of inflation it responds to is given by the following:

$$\Pi_{\mathrm{nmc},t} = \Lambda_t / \Lambda_{t-1} \, \Pi_t.$$

C Paper-and-pencil model variant

This appendix provides the derivations of the simplified model that underlie the paper-and-pencil results of Section 3 in the main text. Appendix C.1 reports the equations of the model after applying the simplifying assumptions spelled out in Section 3.1, first in nonlinear and then in linearized form. Appendix C.2 derives the three-equation representation of the model – in particular the Phillips curve and the IS curve.

C.1 Equilibrium conditions and steady state

This section reports the model of Section 2 under the assumptions spelled out in Section 3.1 of the main text. Appendix C.1.1 reports the non-linear model relations. C.1.2 reports the linearized model relations on which the results will later build. Appendix C.1.3 provides the steady state.

C.1.1 Non-linear equilibrium conditions

All household consumption is consumption of the G-good. Therefore, we drop the explicit reference to G-good consumption. Similarly producer and consumer price indexes coincide. Namely, with $\gamma = 0$, $p_{G,t} = 1$.

Households. The budget constraint of a hand-to-mouth household is given by the following:[33]

$$C_{H,t} = w_t N_{H,t} + T_{H,t}. \tag{C.1}$$

The Euler equation of savers is unchanged

$$C_{S,t}^{-\sigma} = \beta\, \mathbb{E}_t \left\{ C_{S,t+1}^{-\sigma} R_t / \Pi_{t+1} \right\}. \tag{C.2}$$

Labor market. Without wage rigidity and market power, the labor supply first-order conditions of hand-to-mouth and saver households give the following:

$$w_t = \chi C_{H,t}^{\sigma} N_{H,t}^{\varphi}, \tag{C.3}$$

$$w_t = \chi C_{S,t}^{\sigma} N_{S,t}^{\varphi}. \tag{C.4}$$

[33] The budget constraint for savers, $C_{S,t} = w_t N_{S,t} + T_{S,t}$, is redundant by Walras's law. Still, it is worth mentioning that the two budget constraints take an identical form. The reason is that, in equilibrium, because of balanced trade, $b_t = 0$ always.

Production. On the firm side, the production function is unchanged:

$$Y_{G,t} = \left[\alpha E_t^{\frac{\theta-1}{\theta}} + (1-\alpha) N_t^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{C.5}$$

Producer and consumer prices are identical – $p_{G,t} = 1$ – under the assumptions prevailing here. So the Phillips curve takes the following form

$$\begin{aligned}\psi \Pi_t \left(\Pi_t - 1\right) = \varepsilon \left(\Lambda_t - (1+\tau^y)(\varepsilon - 1)/\varepsilon\right) \\ + \psi\beta\, \mathbb{E}_t \left\{ \left(C_{S,t+1}/C_{S,t}\right)^{-\sigma} \Pi_{t+1} \left(\Pi_{t+1} - 1\right) Y_{G,t+1}/Y_{G,t} \right\}.\end{aligned} \tag{C.6}$$

The factor demand relations are unchanged, giving the following:

$$w_t = \Lambda_t (1-\alpha) \left(Y_{G,t}/N_t\right)^{\frac{1}{\theta}}, \tag{C.7}$$

$$p_{E,t} = \Lambda_t \alpha \left(Y_{G,t}/E_t\right)^{\frac{1}{\theta}}. \tag{C.8}$$

Next, absent price subsidies for the E-good, firms' profits are given by the following:

$$D_t = (1+\tau^y) Y_{G,t} - w_t N_t - p_{E,t} E_t. \tag{C.9}$$

Fiscal policy. Transfers to hand-to-mouth households are given by the following:

$$\lambda T_{H,t} = \overline{T}_H + \nu \times \left(D_t - \tau^y Y_{G,t}\right) + \iota \times \vartheta \times p_{E,t} \xi_{E,t}, \tag{C.10}$$

Next, a balanced government budget, implies that transfers to savers are given by the following:

$$(1-\lambda) T_{S,t} = (1-\nu) \times \left(D_t - \tau^y Y_{G,t}\right) + \iota \times (1-\vartheta) \times p_{E,t} \xi_{E,t} - \overline{T}_H. \tag{C.11}$$

Monetary Policy. The Taylor rule is not affected and still given by

$$R_t = R \left(\Pi_t/\Pi\right)^{\phi_\Pi} \exp\{\upsilon_t\}. \tag{C.12}$$

International trade and foreign demand. The assumption of balanced trade implies that $b_t = 0$ and the demand for exports is given by the following

$$X_{G,t} = (1-\iota) \times p_{E,t} \xi_{E,t}. \tag{C.13}$$

There is no need to also specify Y_t^*.

Supply regime for the E-good. This part of the paper does not look at shocks to the price of the E-good or its supply. Therefore, in the *fixed-price* regime

$$p_{E,t} = p_E, \tag{C.14.a}$$

holds for some fixed value of p_E.
In the *fixed-supply* regime

$$\xi_{E,t} = \xi_E, \tag{C.14.b}$$

holds for some fixed value of ξ_E.

Market clearing. The labor market clears if

$$N_t = \lambda N_{H,t} + (1-\lambda) N_{S,t}. \tag{C.15}$$

The market for the E-good clears if

$$\xi_{E,t} = E_t. \tag{C.16}$$

The market for domestic products clears if

$$Y_{G,t} = C_t + X_{G,t}, \tag{C.17}$$

where aggregate consumption is defined as

$$C_t = \lambda C_{H,t} + (1-\lambda) C_{S,t}. \tag{C.18}$$

Summary. Equations (C.1) to (C.18) give 18 equilibrium conditions in the 18 unknowns: $C_{H,t}, C_{S,t}, C_t, D_t, E_t, \Lambda_t, N_{H,t}, N_{S,t}, N_t, \Pi_t, \xi_{E,t}, Y_{G,t}, X_{G,t}, p_{E,t}, R_t, T_{H,t}, T_{S,t}, w_t$.

C.1.2 Linearized equilibrium conditions

This section linearizes the equations in Appendix C.1.1 around the zero-inflation steady state. We denote variables in deviation from the non-stochastic steady state as $\widetilde{X}_t = (X_t - X)$. With this, $\widehat{X}_t = X^{-1}\widetilde{X}_t$ denotes the percentage deviation from steady state. We proceed in the same order as in the previous section.

Households. The hand-to-mouth household's budget constraint (C.1) linearizes to the following:[34]

$$C_H \widehat{C}_{H,t} = wN_H \left(\widehat{w}_t + \widehat{N}_{H,t} \right) + \widetilde{T}_{H,t}. \tag{C.19}$$

The saver's consumption Euler equation (C.20) gives

$$\widehat{C}_{S,t} = \mathbb{E}_t\{\widehat{C}_{S,t+1}\} - \frac{1}{\sigma} \left(\widehat{R}_t - \mathbb{E}_t\{\widehat{\Pi}_{t+1}\} \right). \tag{C.20}$$

Labor market. The households' labor supply conditions (C.3) and (C.4) give

$$\widehat{w}_t = \sigma \widehat{C}_{H,t} + \varphi \widehat{N}_{H,t}, \tag{C.21}$$

$$\widehat{w}_t = \sigma \widehat{C}_{S,t} + \varphi \widehat{N}_{S,t}. \tag{C.22}$$

Production. The production function (C.5) gives

$$\widehat{Y}_{G,t} = (1-\alpha)\,(N/Y_G)^{\frac{\theta-1}{\theta}}\,\widehat{N}_t + \alpha\,(E/Y_G)^{\frac{\theta-1}{\theta}}\,\widehat{E}_t. \tag{C.23}$$

The linearization of Phillips curve (C.24) is

$$\widehat{\Pi}_t = \beta\,\mathbb{E}_t\{\widehat{\Pi}_{t+1}\} + \frac{\varepsilon}{\psi} \Lambda\,\widehat{\Lambda}_t. \tag{C.24}$$

The factor demand relations (C.7) and (C.26) give rise to

$$\widehat{w}_t = \widehat{\Lambda}_t + \frac{1}{\theta} \left(\widehat{Y}_{G,t} - \widehat{N}_t \right), \tag{C.25}$$

$$\widehat{p}_{E,t} = \widehat{\Lambda}_t + \frac{1}{\theta} \left(\widehat{Y}_{G,t} - \widehat{E}_t \right). \tag{C.26}$$

Firms' profits (C.27), in linearized form, are

$$\widetilde{D}_t = (1+\tau^y) Y_G \widehat{Y}_{G,t} - wN \left(\widehat{w}_t + \widehat{N}_t \right) - p_E E \left(\widehat{p}_{E,t} + \widehat{E}_t \right). \tag{C.27}$$

Fiscal policy. Transfers to hand-to-mouth households (see (C.10)) satisfy

$$\lambda \widetilde{T}_{H,t} = \nu \times \left(\widetilde{D}_t - \tau^y Y_G \widehat{Y}_{G,t} \right) + \iota \times \vartheta \times p_E \xi_E \left(\widehat{p}_{E,t} + \widehat{\xi}_{E,t} \right), \tag{C.28}$$

[34] The saver's budget constraint would be

$$C_S \widehat{C}_{S,t} = wN_S \left(\widehat{w}_t + \widehat{N}_{S,t} \right) + \widetilde{T}_{S,t}.$$

while transfers to savers are given by (see (C.11))

$$(1-\lambda)\widetilde{T}_{S,t} = (1-\nu) \times \left(\widetilde{D}_t - \tau^y Y_G \widehat{Y}_{G,t}\right) + \iota \times (1-\vartheta) \times p_E \xi_E \left(\widehat{p}_{E,t} + \widehat{\xi}_{E,t}\right). \quad \text{(C.29)}$$

Monetary policy. The Taylor rule (C.12) gives

$$\widehat{R}_t = \phi_\Pi \widehat{\Pi}_t + \upsilon_t \quad \text{(C.30)}$$

International trade and foreign demand. Foreign's demand for exports (C.13) linearizes to

$$\widetilde{X}_{G,t} = (1-\iota) \times p_E \xi_E \left(\widehat{p}_{E,t} + \widehat{\xi}_{E,t}\right). \quad \text{(C.31)}$$

Supply regime for the E-good. We have the following.
In the *fixed-price* regime (see (C.14.a)) we have

$$\widehat{p}_{E,t} = 0. \quad \text{(C.32.a)}$$

In the *fixed-supply* regime (see (C.14.b)) we have

$$\widehat{\xi}_{E,t} = 0. \quad \text{(C.32.b)}$$

Market clearing. Labor-market clearing (C.15) implies

$$N\widehat{N}_t = \lambda N_H \widehat{N}_{H,t} + (1-\lambda) N_S \widehat{N}_{S,t}. \quad \text{(C.33)}$$

The market clearing condition (C.16) for the E-good gives

$$\widehat{\xi}_{E,t} = \widehat{E}_t. \quad \text{(C.34)}$$

The market clearing for domestic products (C.17) means

$$Y_G \widehat{Y}_{G,t} = C\widehat{C}_t + \widetilde{X}_{G,t}, \quad \text{(C.35)}$$

with (C.18) implying that

$$C\widehat{C}_t = \lambda C_H \widehat{C}_{H,t} + (1-\lambda) C_S \widehat{C}_{S,t}. \quad \text{(C.36)}$$

Summary. Equations (C.20) through (C.36) define 18 equilibrium conditions for the 18 unknowns: $\widehat{C}_{H,t}, \widehat{C}_{S,t}, \widehat{C}_t, \widetilde{D}_t, \widehat{E}_t, \widehat{\Lambda}_t, \widehat{N}_{H,t}, \widehat{N}_{S,t}, \widehat{N}_t, \widehat{\Pi}_t, \widehat{\xi}_{E,t}, \widehat{Y}_{G,t}, \widetilde{X}_{G,t}, \widehat{p}_{E,t}, \widehat{R}_t, \widetilde{T}_{H,t}, \widetilde{T}_{S,t}, \widehat{w}_t$.

C.1.3 Steady state

The simplifying assumptions in Section 3.1 of the main text notably simplify the exposition of the model. The reason is that they simplify the steady state. The Taylor rule (C.12) implies $\Pi = 1$. The consumption Euler equation (C.2) then gives $R = 1/\beta$. Parameter $\overline{T}_H$ can be used to ensure that incomes of savers and hand-to-mouth households are identical in the steady state. $\overline{T}_H$ governs transfer to hand-to-mouth households by (C.10). The corresponding transfers to savers are (C.11). As hand-to-mouth households and savers have the same preferences, they will then also have the same level of consumption and labor supply. For consumption, $C_H = C_S = C$, by (C.1) and (C.18); also reference Footnote 33. For labor supply equations (C.3) and (C.4), parameter χ is used to ensure $N_H = N_S = N = 1$ with N being defined through (C.15). Phillips curve (C.6) along with $\Pi = 1$ and the sales subsidy $(1 + \tau^y) = \epsilon/(\epsilon - 1)$ implies unitary marginal costs, $\Lambda = 1$. In addition, with $N = 1$ and targeting $Y_G = 1$, firms' demand for the E-good in the steady state needs to be $E = 1$; recall (C.5). This will be achieved through setting the price of the E-good, p_E, accordingly (in the fixed-price scenario, (C.14.a)) or the supply $\xi_E = 1$ (in the fixed supply scenario, (C.14.b)). Market clearing for the E-good, (C.16), ensures $\xi_E = E$. Next, with $Y_G = E = \Lambda = 1$, in the targeted steady state, $p_E = \alpha$ from the demand function for E-good inputs, (C.8). The labor demand function (C.7) in turn implies $w = 1 - \alpha$. In this steady state, with $p_E = \alpha$, exports are given by $X_G = (1 - \iota)\alpha$ by (C.13). With this, by (C.17), consumption is $C = Y_G - X_G = 1 - (1 - \iota)\alpha$. Dividends are given by $D = \tau^y = 1/(\epsilon - 1)$ from (C.9).

C.2 Consolidating to the Phillips and IS curves in system (5)

The results in Section 3 of the main text rely on a three-equation representation of the economy: The New Keynesian Phillips curve, the IS equation and the monetary policy rule. Monetary policy rule (4) is the same as (C.30). It remains to derive the Phillips curve representation and the IS curve, both of which are displayed in (5). This is what the current appendix does. Throughout we employ the symmetry of the steady state across households, recall Appendix C.1.3, and the notation introduced in Section 3.1.

C.2.1 Market clearing for E-goods

Market clearing condition (C.34) implies $\widehat{\xi}_{E,t} = \widehat{E}_t$. In the following, we will always use this condition to eliminate $\widehat{\xi}_{E,t}$ from the model.

C.2.2 Goods market clearing

Using foreign demand (C.31), goods market clearing (C.35) can be rewritten as

$$Y_G \widehat{Y}_{G,t} = C\widehat{C}_t + (1-\iota) \times p_E E \left(\widehat{p}_{E,t} + \widehat{E}_t\right). \tag{C.37}$$

In the fixed-price regime of the E-good, (C.32.a) applies; in the fixed-supply regime (C.32.b). Note further that if $\iota = 1$, aggregate consumption and production coincide.

C.2.3 Price and supply of E-good for each supply regime

In the fixed-price regime, by (C.32.a)

$$\Gamma^P_{p_E} = 0, \tag{C.38}$$

and by (C.26)

$$\Gamma^P_E = 1 + \theta\Gamma^P_\Lambda. \tag{C.39}$$

In the fixed-supply regime, by (C.32.b)

$$\Gamma^Q_E = 0, \tag{C.40}$$

and by (C.26)

$$\Gamma^Q_{p_E} = \Gamma^Q_\Lambda + \frac{1}{\theta}. \tag{C.41}$$

C.2.4 Marginal costs

With a symmetric steady state ($C_H = C_S = C$ and $N_H = N_S = N$) one can use (C.33) and (C.36) to aggregate labor supply curves (C.21) and (C.22) to

$$\widehat{w}_t = \sigma\widehat{C}_t + \varphi\widehat{N}_t.$$

Combining this with firms' labor demand (C.25), demand for the E-good (C.26), goods market clearing (C.37), the production function (C.23), and either (C.32.a) or (C.32.b) yields the elasticity of marginal costs to production, that is, Γ_Λ in $\widehat{\Lambda}_t = \Gamma_\Lambda \widehat{Y}_{G,t}$. Necessarily, this depends on the supply regime; namely as follows.[35]

In the fixed-price regime. In the fixed-price regime, and under the restrictions on the steady state from Appendix C.1.3, the cyclical elasticity of marginal costs with respect

[35] These are studied in detail in Proposition 2 in the main text.

to output, Γ_Λ^P, is given by

$$\Gamma_\Lambda^P = \frac{(\sigma+\varphi)(1-\alpha) + \iota\alpha(\sigma+\varphi)}{1+\alpha\theta(\sigma+\varphi) + \iota\frac{\alpha}{1-\alpha}\left[1+\alpha\theta(\sigma+\varphi) - \theta\sigma\right]}. \tag{C.42}$$

As special cases of (C.42) we have: if $\iota = 1$ (a closed economy), $\Gamma_\Lambda^P = \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi}$; if $\iota = 0$ (the E-good owned entirely by Foreign), $\Gamma_\Lambda^P = \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta(\sigma+\varphi)}$.[36]

In the fixed-supply regime. In the fixed supply regime, and under the restrictions on the steady state from Appendix C.1.3, the cyclical elasticity of marginal costs with respect to output, Γ_Λ^Q, is given by

$$\Gamma_\Lambda^Q = \frac{\sigma+\varphi-\frac{\alpha}{\theta}(\sigma-1) + \iota\left[\frac{\alpha}{1-\alpha}\left(\varphi+\frac{\alpha}{\theta}\right) + \sigma\frac{\alpha}{\theta}\right]}{1+(1-\iota)(\sigma-1)\alpha}. \tag{C.43}$$

As special cases of (C.43), we have: if $\iota = 1$ (a closed economy), $\Gamma_\Lambda^Q = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}$.[37] if $\iota = 0$ (the E-good owned entirely by Foreign), $\Gamma_\Lambda^Q = \frac{\sigma+\varphi-(\sigma-1)\alpha/\theta}{1+\alpha(\sigma-1)}$.[38]
Note that in either supply regime Γ_Λ does not depend on household heterogeneity; compare (C.42) and (C.43). At a technical level, this arises from the assumptions on symmetry among the two types of households in steady state.

C.2.5 Derivation of the Phillips curve of the main text

Using that in steady state $\Lambda = 1$, combining Phillips curve (C.24) with the solution for marginal costs in the respective E-good supply regime, (C.42) or (C.43), yields the usual representation of the New Keynesian Phillips curve in terms of production,

$$\widehat{\Pi}_t = \beta\,\mathbb{E}_t\{\widehat{\Pi}_{t+1}\} + \widetilde{\kappa}\,\widehat{Y}_{G,t} \qquad \text{with} \qquad \widetilde{\kappa} = \frac{\varepsilon}{\psi}\Gamma_\Lambda. \tag{C.44}$$

[36] In either case, marginal costs increase in output, $\Gamma_\Lambda^P > 0$. Absent the use of the E-good in production (that is, if $\alpha = 0$) $\Gamma_\Lambda^P = \sigma+\varphi > 0$, the text-book expression for the model with labor only. For a fixed price of the E-good, the elasticity of marginal costs with respect to output *decreases* in α: the more important the fixed-price production factor, the less procyclical are marginal costs. This is intuitive because the price is acyclical by assumption. The elasticity of marginal costs with respect to output *decreases* in θ: the less elastic factor demand, the more procyclical are marginal costs. As $\theta \to \infty$, $\Gamma_\Lambda^P \to 0$.

[37] Here, marginal costs increase in output, $\Gamma_\Lambda^Q > 0$. The elasticity of marginal costs with respect to output also *increases* in α. With fixed supply, the E-good's price is more procyclical than marginal costs, recall (C.41). The larger α, the more do marginal costs inherit this feature. The elasticity of marginal costs with respect to output *decreases* in θ.

[38] Here, the sign of Γ_Λ^Q is ambiguous. $\Gamma_\Lambda^Q > 0$ holds if and only if $\sigma+\varphi > \alpha/\theta(\sigma-1)$. One can show that term $\alpha/\theta(\sigma-1)$ captures the effect on marginal costs of the excess sensitivity of the E-good's price with respect to output (the rise of the E-good's price in excess of movements in the wage). The less substitutable the production factors are (the smaller θ), the larger the excess sensitivity. In turn, the larger the share of the E-good is in production (the larger α), the more this matters for marginal costs. See also the discussion on the wealth effect on labor supply below Proposition 2 in the main text.

Because Γ_Λ is not affected by household heterogeneity, neither is $\widetilde{\kappa}$. Representation (C.44) is the one that we employ in Section 3.2 in the main text; compare (5) there.

In remains to derive the IS-curve representation. This is what we do next.

C.2.6 Production function

Using that in the steady state $Y_G = N = E = 1$, the firms' production function (C.23) relates labor supply to output as

$$\widehat{N}_t = \Gamma_N \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_N = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\Gamma_E, \tag{C.45}$$

where Γ_N is the elasticity of aggregate labor with respect to output. Γ_E follows either (C.39) or (C.40). With this (C.39)

$$\Gamma_N^P = \frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}(1+\theta\Gamma_\Lambda^P). \tag{C.46}$$

And with (C.40)

$$\Gamma_N^Q = \frac{1}{1-\alpha}. \tag{C.47}$$

C.2.7 Wages

Combining firms' labor demand function (C.25) with (C.45) yields

$$\widehat{w}_t = \Gamma_w \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_w = \Gamma_\Lambda + \frac{1}{\theta}\left(1-\Gamma_N\right), \tag{C.48}$$

where Γ_w is the elasticity of wages with respect to output. Again, all these terms depend on the supply regime of the E-good, as follows.
Using (C.48) along with (C.46) gives

$$\Gamma_w^P = \frac{1}{1-\alpha}\Gamma_\Lambda^P, \tag{C.49}$$

so that wages are unambiguously more cyclical than marginal costs.
Similarly, using (C.48) along with (C.47) gives

$$\Gamma_w^Q = \Gamma_\Lambda^Q - \frac{\alpha}{\theta(1-\alpha)}, \tag{C.50}$$

so that wages are less cyclical than marginal costs.[39]

[39] The text below Propositions 3 and 4 in the main text discusses how the elasticity of aggregate demand to production is shaped by the different elasticities of wages in the two supply regimes. As an aside, using (C.26) and (C.50), one can derive a measure of the "excess sensitivity" of the E-good price to output

C.2.8 Dividends and lump-sum income

Using the steady state in (C.27) gives

$$\widetilde{D}_t = (1+\tau^y)\widehat{Y}_{G,t} - (1-\alpha)\left(\widehat{w}_t + \widehat{N}_t\right) - \alpha\left(\widehat{p}_{E,t} + \widehat{E}_t\right),$$

or, using the production function (C.23),

$$\widetilde{D}_t = \tau^y\widehat{Y}_{G,t} - (1-\alpha)\widehat{w}_t - \alpha\widehat{E}_t.$$

Next, combining the factor demand functions (C.25) and (C.26) with the production function (C.23), shows that marginal costs are given by

$$\widehat{\Lambda}_t = (1-\alpha)\widehat{w}_t + \alpha\widehat{E}_t,$$

so that

$$\widetilde{D}_t = \tau^y\widehat{Y}_{G,t} - \widehat{\Lambda}_t,$$

The semi elasticity of dividends with respect to output, Γ_D, thereby is

$$\widetilde{D}_t = \Gamma_D\widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_D = \tau^y - \Gamma_\Lambda. \tag{C.51}$$

As per transfers, using (C.51) and (C.28) along with the steady state implies

$$\widetilde{T}_{H,t} = \Gamma_{T_H}\widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_{T_H} = \frac{\nu}{\lambda}\times(\Gamma_D - \tau^y) + \iota\times\frac{\vartheta}{\lambda}\times\alpha\left(\Gamma_{p_E} + \Gamma_E\right), \tag{C.52}$$

where Γ_{T_H} is the semi-elasticity of a hand-to-mouth household's lump-sum income with respect to output. Next, using (C.51) and (C.29) along with the steady state implies

$$\widetilde{T}_{S,t} = \Gamma_{T_S}\widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_{T_S} = \frac{(1-\nu)}{(1-\lambda)}\times(\Gamma_D - \tau^y) + \iota\times\frac{(1-\vartheta)}{(1-\lambda)}\times\alpha\left(\Gamma_{p_E} + \Gamma_E\right), \tag{C.53}$$

where Γ_{T_S} is the semi-elasticity of a saver household's lump-sum income.[40]

relative to the sensitivity of wages, namely $\Gamma^Q_{P_E} - \Gamma^Q_w = \frac{1}{\theta}\frac{1}{1-\alpha} > 0$.

[40]The text surrounding Propositions 3 and 4 in the main text discusses how the elasticity of aggregate demand to production is shaped by the different elasticities of the components of lump-sum income across the supply regimes of the E-good.

C.2.9 Aggregate consumption

Using the steady-state values of Appendix C.1.3 and the goods-market clearing condition (C.37), one can define the elasticity of aggregate consumption with respect to output as

$$\widehat{C}_t = \Gamma_C \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_C = \frac{1-(1-\iota)\alpha\left(\Gamma_{p_E}+\Gamma_E\right)}{1-\alpha+\iota\alpha}, \tag{C.54}$$

where, as usual, Γ_C depends on the supply regime.[41]

C.2.10 Hand-to-mouth households' consumption

Start with hand-to-mouth households' budget (C.19). In this, substitute for employment from the households' labor supply first-order condition (C.21). Next, use the steady state results from Appendix C.1.3 to get that

$$\widehat{C}_{H,t} = \Gamma_{C_H} \widehat{Y}_{G,t}$$
$$\text{with} \qquad \Gamma_{C_H} = \frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(\sigma+\varphi)+\iota\alpha\varphi}\Gamma_w + \frac{\varphi}{(1-\alpha)(\sigma+\varphi)+\iota\alpha\varphi}\Gamma_{T_H}, \tag{C.55}$$

where Γ_{C_H} is the elasticity of a hand-to-mouth household's consumption with respect to output. This, too, depends on the supply regime for the E-good of course.

The proofs of Appendix D will look at one of two special cases. Either $\iota = 0$ or $\iota = 1$, and at the two supply regimes. The following paragraphs prepare the ground for these proofs by providing expressions for the Γ_{C_H} of (C.55) in these cases. The reader who is only interested in knowing that there is an IS-curve representation of the demand side, can skip these special cases and jump to Appendix C.2.11.

Special cases in the fixed-price regime. We look at $\Gamma^P_{C_H}$ in the two polar cases for ι. The expressions below result from substituting in (C.55) for Γ^P_w from (C.49) and for $\Gamma^P_{T_H}$ from (C.52). Next, substitute for $\Gamma^P_{p_E}$ from (C.38) and for Γ^P_E from (C.39).

In case that $\iota = 1$ (a closed economy), then (C.55) gives

$$\Gamma^P_{C_H} = \frac{1+\varphi}{(1-\alpha)\sigma+\varphi}\Gamma^P_\Lambda + \frac{\varphi}{(1-\alpha)\sigma+\varphi}\left[-\frac{\nu}{\lambda}\times\Gamma^P_\Lambda + \frac{\vartheta}{\lambda}\times\alpha\left(1+\theta\Gamma^P_\Lambda\right)\right], \tag{C.56}$$

where $\Gamma^P_\Lambda = (1-\alpha)(\sigma+\varphi)/(1+\alpha\theta\varphi)$, see (C.42) with $\iota = 1$.

The first term reflects the cyclicality of labor income. Ownership of dividends (ν) makes hand-to-mouth households' income (ceteris paribus) more countercyclical, while ownership of E-good revenues (ϑ) makes hand-to-mouth households' income (ceteris paribus) more procyclical.

[41] If $\iota = 1$ (a closed economy), consumption and production coincide. If $\iota = 0$ (the E-good is owned externally only), consumption can potentially *fall* even though production rises. This is because consumption is value-added: when the E-good's price increases, the value-added share of production falls.

Next, focus on the case $\iota = 0$ (the E-good is owned externally only), then (C.55) gives

$$\Gamma^P_{C_H} = \frac{(1+\varphi)/(1-\alpha)}{\sigma+\varphi}\Gamma^P_\Lambda - \frac{\nu}{\lambda} \times \frac{\varphi/(1-\alpha)}{\sigma+\varphi}\Gamma^P_\Lambda, \tag{C.57}$$

where $\Gamma^P_\Lambda = (1-\alpha)(\sigma+\varphi)/(1+\alpha\theta(\sigma+\varphi))$, see (C.42) with $\iota = 0$.

Special cases in the fixed-supply regime. Again, we look at $\Gamma^Q_{C_H}$ in the two polar cases for ι. The expressions below result from substituting in (C.55) for Γ^Q_w from (C.50) and for $\Gamma^P_{T_H}$ from (C.52). Next, substitute for $\Gamma^Q_{p_E}$ from (C.41) and for Γ^Q_E from (C.40). In case that $\iota = 1$ (a closed economy), then (C.55) gives

$$\Gamma^Q_{C_H} = 1 + \varphi + \frac{\varphi}{(1-\alpha)\sigma+\varphi}\left[-\frac{\nu}{\lambda}\Gamma^Q_\Lambda + \frac{\vartheta}{\lambda}\alpha\left(\Gamma^Q_\Lambda + \frac{1}{\theta}\right)\right], \tag{C.58}$$

where $\Gamma^Q_\Lambda = \sigma + \varphi/(1-\alpha) + \alpha/\theta \cdot 1/(1-\alpha)$, see (C.43) for $\iota = 1$.
Next, focus on the case $\iota = 0$ (the E-good is owned externally only), then (C.55) gives

$$\Gamma^Q_{C_H} = \frac{1+\varphi}{\sigma+\varphi}\left(\Gamma^Q_\Lambda - \frac{\alpha/\theta}{1-\alpha}\right) - \frac{\nu}{\lambda} \times \frac{\varphi}{\sigma+\varphi}\frac{1}{1-\alpha}\Gamma^Q_\Lambda, \tag{C.59}$$

where $\Gamma^Q_\Lambda = \frac{\sigma+\varphi-(\sigma-1)\alpha/\theta}{1+\alpha(\sigma-1)}$, see (C.43) for $\iota = 0$.

C.2.11 Savers' consumption

So as to relate saver's consumption to output, use the definition of aggregate consumption, (C.36), along with Γ_C from (C.54) and Γ_{C_H} from (C.55). Then a saver's consumption can be expressed as

$$\widehat{C}_{S,t} = \Gamma_{C_S}\widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_{C_S} = \frac{\Gamma_C - \lambda\Gamma_{C_H}}{1-\lambda}, \tag{C.60}$$

where we have used the steady-state values spelled out in Appendix C.1.3. Note that the sign of Γ_{C_S} is fully determined by $\Gamma_C - \lambda\Gamma_{C_H}$. In a closed economy ($\iota = 1$), $\Gamma_C = 1$, so that the numerator is given by $1 - \lambda\Gamma_{C_H}$, as in the standard TANK model. The innovation of our exercise is to account for how the supply regime of the E-good affects Γ_{C_H}. In the other extreme case, if the E-good is owned externally only $\iota = 0$, aggregate demand no longer is given by consumption only, so that $\Gamma_C \neq 1$.

C.2.12 Derivation of the IS curve in the main text

Combining savers' Euler equation (C.20) with the expression for (C.60) (and focusing on parameter constellations in which $\Gamma_{C_S} \neq 0$) yields the IS curve

$$\widehat{Y}_{G,t} = \mathbb{E}_t\{\widehat{Y}_{G,t+1}\} - \frac{1}{\widetilde{\sigma}}\left(\widehat{R}_t - \mathbb{E}_t\{\widehat{\Pi}_{t+1}\}\right) \qquad \text{with} \qquad \widetilde{\sigma} = \sigma\,\Gamma_{C_S}. \tag{C.61}$$

Note that $\widetilde{\sigma} > 0$ if and only if $\Gamma_{C_S} > 0$. This is the case if and only if $\Gamma_C - \lambda\Gamma_{C_H} > 0$. Hence, the IS curve can invert when $\lambda\Gamma_{C_H} > \Gamma_C$, or, in words, when the elasticity of hand-to-mouth households' income towards production is greater than their inverse weight in the economy. This is as in the "standard" TANK model. What we emphasize is how the distribution of incomes in the two supply regimes affects both Γ_C and Γ_{C_H}. Thus, here, the IS curve can invert if in a standard TANK model it would not, and *vice versa*.

Indeed, the Foreign economy is a source of demand for goods produced in Home. Thus, even if households in Home would not be heterogeneous, the IS curve may invert. Proposition E.1 in Appendix E zooms in on this case.

Representation (C.61) is the one that we employ in Section 3.2 in the main text; compare equation (5) there.

D Proofs of paper-and-pencil results

This appendix provides the proofs for all the propositions in the paper. Appendix D.1 provides the proof of Proposition 1, Appendix D.2 of Proposition 2, Appendix D.3 of Proposition 3, and Appendix D.4 of Proposition 4.

D.1 Proof of Proposition 1

This appendix provides the proof for Proposition 1. The proof is straightforward and the steps well known in the New Keynesian literature. The model is given by equations (5), repeated here for convenience,

$$\widehat{\Pi}_t = \beta\,\mathbb{E}_t\{\widehat{\Pi}_{t+1}\} + \widetilde{\kappa}\,\widehat{Y}_{G,t} \quad \text{and} \quad \widehat{Y}_{G,t} = \mathbb{E}_t\{\widehat{Y}_{G,t+1}\} - \frac{1}{\widetilde{\sigma}}\left(\widehat{R}_t - \mathbb{E}_t\{\widehat{\Pi}_{t+1}\}\right). \tag{D.1}$$

The convolute parameters $\widetilde{\kappa}$ and $\widetilde{\sigma}$ depend on the precise specification of the model, as discussed extensively in Section 3 and Appendix C.2. Take them as given.
Write the model in Blanchard and Kahn (1980) form as

$$\begin{bmatrix} 1 & 1/\widetilde{\sigma} \\ 0 & \beta \end{bmatrix} \times \mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \phi_\Pi/\widetilde{\sigma} \\ -\widetilde{\kappa} & 1 \end{bmatrix} \times \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix},$$

or, alternatively

$$\mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}} & \frac{1}{\widetilde{\sigma}}\phi_\Pi - \frac{1}{\beta}\frac{1}{\widetilde{\sigma}} \\ -\frac{1}{\beta}\widetilde{\kappa} & \frac{1}{\beta} \end{bmatrix}}_{:=A} \times \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix}$$

There are two nonpredetermined variables. So there will always be bounded equilibria. There is a locally unique bounded equilibrium iff either (cf. Woodford, 2003, p. 670):

- Case a): $det(A) > 1$, $det(A) - tr(A) > -1$ and $det(A) + tr(A) > -1$, or
- Case b): $det(A) - tr(A) < -1$ and $det(A) + tr(A) < -1$.

Here, $det(A) = \left[\frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\phi_\Pi\right]$ and $tr(A) = \left[1 + \frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\right]$.

Proof of the proposition's item 1). Suppose that $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} > 0$. Then the determinacy conditions are as in the standard New Keynesian model. More in detail, $det(A) > 1$ and $tr(A) > 0$, so that Case a) applies. The condition that may bind is $det(A) - tr(A) > -1$, which leads to the conventional determinacy condition $\phi_\Pi > 1$.

Proof of the proposition's item 1) c'td. Suppose that $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} < 0$. Again, in this case $det(A) > 1$ for any $\phi_\Pi > 0$. Thus, we need to check Case a) again. $tr(A) > 0$, so that $det(A) + tr(A) > -1$ always. So, what we need for determinacy is $det(A) - tr(A) > -1$. Or, once more, $\phi_\Pi > 1$.

Proof of the proposition's item 2). Suppose that $\widetilde{\sigma} < 0$, $\widetilde{\kappa} > 0$. In this case, *two* determinacy regions can arise. Focus on the set of conditions for case a) first. $det(A) = \left[\frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\phi_\Pi\right] > 1$ can be achieved by setting $\phi_\Pi < \frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1)$, where $\frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1) > 0$ as $\frac{\widetilde{\sigma}}{\widetilde{\kappa}} < 0$. The second condition can be achieved by setting $\phi_\Pi < 1$. Finally, the third condition can be achieved by setting $\phi_\Pi < -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1$. Hence, this determinacy region exists if there is a $\phi_\Pi \geq 0$ such that

$$\phi_\Pi < \min\left(\frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1),\ 1,\ -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right)$$

Note that, the assumptions of Section 3.1 include that $\beta \to 1$. Then, this determinacy region disappears.
Focus on the set of conditions for case b) next. For $det(A) - tr(A) < -1$, we need $\frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}[\phi_\Pi - 1] - 1 < -1$, meaning $\phi_\Pi > 1$. For $det(A) + tr(A) < -1$, we need $1 + \frac{2}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}(\phi_\Pi + 1) < -1$, meaning $\phi_\Pi > -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1$. So that both $det(A) \pm tr(A) < -1$, therefore we need

$$\phi_\Pi > \max\left(1, -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right),$$

or for $\beta \to 1$, $\phi_\Pi > \max(1, -4\widetilde{\sigma}/\widetilde{\kappa} - 1)$. This is the cutoff mentioned in Proposition 1, equation (7).

Proof of the proposition's item 2), c'td. Suppose that $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} < 0$. In all the derivations in the previous paragraph all $\widetilde{\sigma}$ and $\widetilde{\kappa}$ only appear in the form of their $\widetilde{\kappa}/\widetilde{\sigma}$ in all derivations. Thus the case $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} < 0$ has the same determinacy regions as the case $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$. □

D.2 Proof of Proposition 2

This appendix proves Proposition 2. The results build on the derivations in Appendix C.2. By (C.44), for given parameters ϵ and ψ, the slope of the Phillips curve only depends on Γ_Λ, the general elasticity of marginal costs with respect to output. (C.42) presents Γ_Λ^P, that is the elasticity for the case of a fixed price. (C.43) presents Γ_Λ^Q, the elasticity amid a fixed supply of the E-good.

Proof of the proposition's item 1. Under the assumption of domestic ownership ($\iota = 1$), the terms simplify to

$$\Gamma_\Lambda^Q = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}$$

and

$$\Gamma_\Lambda^P = \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi}.$$

Note that both of these terms are strictly positive. Straightforward algebra then shows that $\Gamma_\Lambda^Q > \Gamma_\Lambda^P$ is equivalent to

$$\alpha(1-\alpha)\sigma + \frac{\alpha}{\theta} + \alpha(1-\alpha)\theta\varphi\sigma + \alpha\theta\varphi^2 + 2\alpha\varphi > 0,$$

which holds for all admissible parameter values, as $\alpha \in (0,1)$, $\theta > 0$, $\sigma > 0$, $\varphi > 0$. This provides the proof of the first item.

Proof of the proposition's item 2. Under the assumption of complete foreign ownership of the E-good ($\iota = 0$), the expressions for Γ_Λ simplify to, respectively,

$$\Gamma_\Lambda^Q = \frac{\sigma + \varphi - \frac{\alpha}{\theta}(\sigma - 1)}{1 + \alpha(\sigma - 1)} \tag{D.2}$$

and

$$\Gamma_\Lambda^P = \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta(\sigma+\varphi)}. \tag{D.3}$$

Note that Γ_Λ^P is strictly positive as stated in the proposition. Straightforward algebra shows that $\Gamma_\Lambda^Q > \Gamma_\Lambda^P$ is equivalent to $\sigma + \varphi > (\sigma - 1)/\theta$. This completes the proof of Proposition 2. □

D.3 Proof of Proposition 3

This appendix proves Proposition 3. The results build on the derivations in Appendix C.2. The proposition is concerned with how the supply regime for the E-good shapes Γ_{C_S}. Γ_{C_S} matters because by (C.61), it determines the slope of the IS curve.

The proposition looks at the case that $\iota = 1$, that is, all of the supply of the E-good is owned domestically. In that case $\Gamma_C = 1$ by (C.54). By equation (C.60), then $\Gamma_{C_S} = (1 - \lambda\Gamma_{C_H})/(1 - \lambda)$. What matters is Γ_{C_H}.

The proposition looks at three cases for ownership of firms and the E-good. Namely, 1. $\nu = \vartheta = 0$, 2. $\nu = 0$, $\vartheta > 0$, and 3. $\nu = \vartheta > 0$. In terms of notation, a superscript $i \in \{(1), (2), (3)\}$ refers to these cases of ownership. So that $\Gamma_{C_H}^{P,(1)}$, for example, refers to the output elasticity of a hand-to-mouth household's consumption when in the fixed-price regime and $\nu = \vartheta = 0$ (case 1.)

Appendix C.2.10 spells out Γ_{C_H} in the two supply regimes and for $\iota = 1$. Next, we spell out $\Gamma_{C,H}^{\cdot,i}$ for each supply regime, then we compare across the two supply regimes.

Fixed-price regime for the E-good. For the fixed-price regime, $\Gamma_{C_H}^P$ is given by (C.56), where Γ_Λ^P is given by (C.42), both restated here for convenience,

$$\Gamma_{C_H}^P = \frac{1+\varphi}{(1-\alpha)\sigma+\varphi}\Gamma_\Lambda^P + \frac{\varphi}{(1-\alpha)\sigma+\varphi}\left[-\frac{\nu}{\lambda}\times\Gamma_\Lambda^P + \frac{\vartheta}{\lambda}\times\alpha\left(1+\theta\Gamma_\Lambda^P\right)\right],$$
$$\Gamma_\Lambda^P = \frac{(1-\alpha)\left(\sigma+\varphi\right)}{1+\alpha\theta\varphi}.$$

Denote with a number-superscript $i \in \{(1), (2), (3)\}$ to $\Gamma_{C_H}^{P,i}$ the respective cases. Then, simplifying gives for

$$\begin{aligned}
\nu = \vartheta = 0: &\qquad \Gamma_{C_H}^{P,(1)} = (1+\varphi)\frac{(1-\alpha)(\sigma+\varphi)}{\left((1-\alpha)\sigma+\varphi\right)(1+\alpha\theta\varphi)},\\
\nu = 0,\ \vartheta > 0: &\qquad \Gamma_{C_H}^{P,(2)} = \Gamma_{C_H}^{P,(1)} + \frac{\vartheta}{\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[1+\frac{\theta(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi}\right],\\
\nu = \vartheta > 0: &\qquad \Gamma_{C_H}^{P,(3)} = \Gamma_{C_H}^{P,(1)} + \frac{\vartheta}{\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[1-\frac{(1-\alpha\theta)(1-\alpha)(\sigma+\varphi)}{\alpha(1+\alpha\theta\varphi)}\right].
\end{aligned}$$

Then, as $\Gamma_{C_S}^P = (1-\lambda\Gamma_{C_H}^P)/(1-\lambda)$, we have for

$$\begin{aligned}
\nu = \vartheta = 0: &\quad \Gamma_{C_S}^{P,(1)} = \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}(1+\varphi)\frac{(1-\alpha)(\sigma+\varphi)}{\left((1-\alpha)\sigma+\varphi\right)(1+\alpha\theta\varphi)},\\
\nu = 0,\ \vartheta > 0: &\quad \Gamma_{C_S}^{P,(2)} = \Gamma_{C_S}^{P,(1)} - \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[1+\frac{\theta(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi}\right],\\
\nu = \vartheta > 0: &\quad \Gamma_{C_S}^{P,(3)} = \Gamma_{C_S}^{P,(1)} - \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[1-\frac{(1-\alpha\theta)(1-\alpha)(\sigma+\varphi)}{\alpha(1+\alpha\theta\varphi)}\right].
\end{aligned}$$

Fixed-supply regime for the E-good. For the fixed-supply regime, $\Gamma^{Q}_{C_H}$ is given by (C.58), where Γ^{Q}_{Λ} is given by (C.43), both restated here for convenience,

$$\Gamma^{Q}_{C_H} = 1 + \varphi + \frac{\varphi}{(1-\alpha)\sigma + \varphi}\left[-\frac{\nu}{\lambda}\Gamma^{Q}_{\Lambda} + \frac{\vartheta}{\lambda}\alpha\left(\Gamma^{Q}_{\Lambda} + \frac{1}{\theta}\right)\right],$$
$$\Gamma^{Q}_{\Lambda} = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}.$$

Denote with a number-superscript $i \in \{(1),(2),(3)\}$ to $\Gamma^{Q,i}_{C_H}$ the respective cases. Then, simplifying gives for

$$\begin{aligned}
\nu = \vartheta = 0: \qquad & \Gamma^{Q,(1)}_{C_H} = 1 + \varphi,\\
\nu = 0,\ \vartheta > 0: \qquad & \Gamma^{Q,(2)}_{C_H} = \Gamma^{Q,(1)}_{C_H} + \frac{\vartheta}{\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[\frac{(1-\alpha)\sigma + \varphi + 1/\theta}{1-\alpha}\right],\\
\nu = \vartheta > 0: \qquad & \Gamma^{Q,(3)}_{C_H} = \Gamma^{Q,(1)}_{C_H} + \frac{\vartheta}{\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[-\frac{(1-\alpha)\sigma + \varphi}{\alpha}\right].
\end{aligned}$$

Then, as $\Gamma^{Q}_{C_S} = (1-\lambda\Gamma^{Q}_{C_H})/(1-\lambda)$, we have for

$$\begin{aligned}
\nu = \vartheta = 0: \qquad & \Gamma^{Q,(1)}_{C_S} = \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}(1+\varphi),\\
\nu = 0,\ \vartheta > 0: \qquad & \Gamma^{Q,(2)}_{C_S} = \Gamma^{Q,(1)}_{C_S} - \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[\frac{(1-\alpha)\sigma + \varphi + 1/\theta}{1-\alpha}\right],\\
\nu = \vartheta > 0: \qquad & \Gamma^{Q,(3)}_{C_S} = \Gamma^{Q,(1)}_{C_S} - \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\left[-\frac{(1-\alpha)\sigma + \varphi}{\alpha}\right].
\end{aligned}$$

Comparison across regimes. We go over the cases one by one.
<u>Consider case 1</u>, ($\nu = \vartheta = 0$):

$$\Gamma^{Q,(1)}_{C_S} - \Gamma^{P,(1)}_{C_S} = \frac{\lambda}{1-\lambda}(1+\varphi)\left(\Delta^{(1)} - 1\right) < 0,$$

as $\Delta^{(1)} := \frac{(1-\alpha)(\sigma+\varphi)}{((1-\alpha)\sigma+\varphi)(1+\alpha\theta\varphi)} < 1$. Thus, $\Gamma^{Q,(1)}_{C_S} - \Gamma^{P,(1)}_{C_S} < 0$, as stated in item 1 of the proposition.
<u>Consider case 2</u>, ($\nu = 0$, $\vartheta > 0$):

$$\Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S} = \Gamma^{Q,(1)}_{C_S} - \Gamma^{P,(1)}_{C_S} + \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\Delta^{(2)} < \Gamma^{Q,(1)}_{C_S} - \Gamma^{P,(1)}_{C_S},$$

as $\Delta^{(2)} := 1 + \frac{\theta(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta\varphi} - \frac{(1-\alpha)\sigma+\varphi+1/\theta}{1-\alpha} < 0$. The sign of this can be established taking into account that $\sigma > 0, \alpha \in (0,1), \varphi > 0$ and, especially, $\theta \in (0,1)$. Thus, $\Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S} < \Gamma^{Q,(1)}_{C_S} - \Gamma^{P,(1)}_{C_S}$, as was the claim in the proposition's item 2.

Consider case 3, $(\nu = \vartheta > 0)$:

$$\Gamma_{C_S}^{Q,(3)} - \Gamma_{C_S}^{P,(3)} = \Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)} + \frac{\vartheta}{1-\lambda}\frac{\alpha\varphi}{(1-\alpha)\sigma+\varphi}\Delta^{(3)} > \Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)},$$

as $\Delta^{(3)} := 1 - \frac{(1-\alpha\theta)(1-\alpha)(\sigma+\varphi)}{\alpha(1+\alpha\theta\varphi)} + \frac{(1-\alpha)\sigma+\varphi}{\alpha} > 0$. Thus, $\Gamma_{C_S}^{Q,(3)} - \Gamma_{C_S}^{P,(3)} > \Gamma_{C_S}^{Q,(1)} - \Gamma_{C_S}^{P,(1)}$, as was the claim in the proposition's item 3.

This concludes the proof of Proposition 3. □

D.4 Proof of Proposition 4

This appendix proves Proposition 4. The outline of the proof is the same as for Proposition 3. Once more, the results build on the derivations in Appendix C.2. The proposition is concerned with how the supply regime for the E-good shapes Γ_{C_S}. Γ_{C_S} matters because by (C.61), it determines the slope of the IS curve.

The proposition looks at the case of $\iota = 0$, that is, all of the supply of the E-good is owned externally. By (C.60) $\Gamma_{C_S} = (\Gamma_C - \lambda\Gamma_{C_H})/(1-\lambda)$. What matters, therefore, are both Γ_C and Γ_{C_H}.

The proposition looks at three cases for the ownership of firms. Namely, 1. $\nu = \lambda$, 2. $\nu = 0$, and 3. $\nu > 0$. In terms of notation, a superscript $i \in \{(1),(2),(3)\}$ refers to these cases of ownership. So that $\Gamma_{C_H}^{P,(1)}$, for example, refers to the output elasticity of a hand-to-mouth household's consumption in the fixed-price regime and $\nu = \lambda$ (case 1.)

We first spell out the respective elasticities for each supply regime separately. Thereafter, we compare across the two supply regimes.

Fixed-price regime for the E-good For the fixed-price regime, Γ_C^P is given by (C.54) and $\Gamma_{C_H}^P$ is given by (C.57), where Γ_Λ^P is given by (C.42) and Γ_E^P is given by (C.39), all restated here for convenience,

$$\begin{aligned}
\Gamma_C^P &= \frac{1-\alpha\Gamma_E^P}{1-\alpha},\\
\Gamma_{C_H}^P &= \frac{(1+\varphi)/(1-\alpha)}{\sigma+\varphi}\Gamma_\Lambda^P - \frac{\nu}{\lambda}\frac{\varphi/(1-\alpha)}{\sigma+\varphi}\Gamma_\Lambda^P,\\
\Gamma_\Lambda^P &= \frac{(1-\alpha)(\sigma+\varphi)}{1+\alpha\theta(\sigma+\varphi)},\\
\Gamma_E^P &= 1+\theta\Gamma_\Lambda^P.
\end{aligned}$$

Note, first, that for all the cases $i \in \{(1),(2),(3)\}$,

$$\Gamma_C^P = \frac{1}{1+\alpha\theta(\sigma+\varphi)} > 0.$$

Denote with a number-superscript $i \in \{1,2,3\}$ to $\Gamma_{C_H}^{P,i}$ the respective case. Then, simplifying gives for

$$\begin{aligned}
\nu = \lambda: &\qquad \Gamma_{C_H}^{P,(1)} = \frac{1}{1+\alpha\theta(\sigma+\varphi)} = \Gamma_C^P,\\
\nu = 0: &\qquad \Gamma_{C_H}^{P,(2)} = \frac{1+\varphi}{1+\alpha\theta(\sigma+\varphi)},\\
\nu > 0: &\qquad \Gamma_{C_H}^{P,(3)} = \Gamma_{C_H}^{P,(2)} - \frac{\nu}{\lambda}\frac{\varphi}{1+\alpha\theta(\sigma+\varphi)}.
\end{aligned}$$

Then, as $\Gamma^P_{C_S} = (\Gamma^P_C - \lambda\Gamma^P_{C_H})/(1-\lambda)$, we have for

$$\begin{aligned}
\nu = \lambda: \qquad & \Gamma^{P,(1)}_{C_S} = \Gamma^P_C, \\
\nu = 0: \qquad & \Gamma^{P,(2)}_{C_S} = \frac{1}{1-\lambda}\Gamma^P_C - \frac{\lambda}{1-\lambda}\frac{1+\varphi}{1+\alpha\theta(\sigma+\varphi)}, \\
\nu > 0: \qquad & \Gamma^{P,(3)}_{C_S} = \Gamma^{P,(2)}_{C_S} + \frac{\nu}{1-\lambda}\frac{\varphi}{1+\alpha\theta(\sigma+\varphi)}.
\end{aligned}$$

Focus on the case that $\nu = \lambda$. Because $\Gamma^P_C = \Gamma^{P,(1)}_{C_H} > 0$ and $\lambda \in [0,1)$, by (C.60) we also have that $\Gamma^{P,(1)}_{C_S} > 0$, the first part of the statement in (17) in Proposition 4.

Fixed-supply regime for the E-good. For the fixed-supply regime, Γ^Q_C is given by (C.54), and $\Gamma^Q_{C_H}$ is given by (C.59), where Γ^Q_Λ is given by (C.43) and $\Gamma^Q_{p_E}$ is given by (C.41), all restated here for convenience,

$$\begin{aligned}
\Gamma^Q_C &= \frac{1-\alpha\Gamma^Q_{p_E}}{1-\alpha}, \\
\Gamma^Q_{C_H} &= \frac{1+\varphi}{\sigma+\varphi}\left(\Gamma^Q_\Lambda - \frac{\alpha/\theta}{1-\alpha}\right) - \frac{\nu}{\lambda}\frac{\varphi}{\sigma+\varphi}\frac{1}{1-\alpha}\Gamma^Q_\Lambda, \\
\Gamma^Q_\Lambda &= \frac{\sigma+\varphi-(\sigma-1)\alpha/\theta}{1+\alpha(\sigma-1)}, \\
\Gamma^Q_{p_E} &= \Gamma^Q_\Lambda + 1/\theta.
\end{aligned}$$

Note, first, that for all the cases $i \in \{(1),(2),(3)\}$,

$$\Gamma^Q_C = \frac{1-\alpha\,[1+\varphi+1/\theta]}{(1-\alpha)(1+\alpha(\sigma-1))}.$$

Denote with a number-superscript $i \in \{1,2,3\}$ to $\Gamma^{P,(i)}_{C_H}$ the respective case. Then, simplifying gives for

$$\begin{aligned}
\nu = \lambda: \qquad & \Gamma^{Q,(1)}_{C_H} = \frac{1-\alpha\,[1+\varphi+1/\theta]}{(1-\alpha)(1+\alpha(\sigma-1))} = \Gamma^Q_C, && \text{(D.4)} \\
\nu = 0: \qquad & \Gamma^{Q,(2)}_{C_H} = \frac{1+\varphi}{\sigma+\varphi}\left(\frac{(1-\alpha)(\sigma+\varphi)-\sigma\,\alpha/\theta}{(1-\alpha)(1+\alpha(\sigma-1))}\right), \\
\nu > 0: \qquad & \Gamma^{Q,(3)}_{C_H} = \Gamma^{Q,(2)}_{C_H} - \frac{\nu}{\lambda}\frac{\varphi}{\sigma+\varphi}\left(\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{(1-\alpha)(1+\alpha(\sigma-1))}\right).
\end{aligned}$$

Then, as $\Gamma^Q_{C_S} = (\Gamma^Q_C - \lambda \Gamma^Q_{C_H})/(1-\lambda)$, we have for

$$\begin{aligned}
\nu = \lambda: &\qquad \Gamma^{Q,(1)}_{C_S} = \Gamma^Q_C, \\
\nu = 0: &\qquad \Gamma^{Q,(2)}_{C_S} = \frac{1}{1-\lambda}\Gamma^Q_C - \frac{\lambda}{1-\lambda}\frac{1+\varphi}{\sigma+\varphi}\left(\frac{(1-\alpha)(\sigma+\varphi)-\sigma\,\alpha/\theta}{(1-\alpha)(1+\alpha(\sigma-1))}\right), \\
\nu > 0: &\qquad \Gamma^{Q,(3)}_{C_S} = \Gamma^{Q,(2)}_{C_S} + \frac{\nu}{1-\lambda}\frac{\varphi}{\sigma+\varphi}\left(\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{(1-\alpha)(1+\alpha(\sigma-1))}\right).
\end{aligned} \tag{D.5}$$

Comparison across regimes. We go over the cases one by one.
Consider case 1 ($\nu = \lambda$) and prove (17) in Proposition 4: That $\Gamma^{P,(1)}_{C_S} > 0$ was shown above. Using the results given above, straightforward algebra also shows that $\Gamma^{P,(1)}_{C_S} > \Gamma^{Q,(1)}_{C_S}$ under the parameter restrictions entertained in this paper. This concludes the proof of (17) in the proposition.
Consider case 2 ($\nu = 0$) and case 3 ($\nu > 0$). In the following expressions, the value of $\nu^{(3)}$ is the value under case 3, so that $\nu > 0$. To prove the statement in (18), postulate the inequality

$$\left(\Gamma^{Q,(3)}_{C_S} - \Gamma^{P,(3)}_{C_S}\right) < \left(\Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S}\right).$$

So that

$$0 > \left(\Gamma^{Q,(3)}_{C_S} - \Gamma^{P,(3)}_{C_S}\right) - \left(\Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S}\right) = \left(\Gamma^{Q,(3)}_{C_S} - \Gamma^{Q,(2)}_{C_S}\right) - \left(\Gamma^{P,(3)}_{C_S} - \Gamma^{P,(2)}_{C_S}\right)$$

Using the expressions above, this gives

$$0 > \frac{\nu^{(3)}}{1-\lambda}\frac{\varphi}{\sigma+\varphi}\left(\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{(1-\alpha)(1+\alpha(\sigma-1))}\right) - \frac{\nu^{(3)}}{1-\lambda}\frac{\varphi}{1+\alpha\theta(\sigma+\varphi)}.$$

Thus, $\Gamma^{Q,(3)}_{C_S} - \Gamma^{P,(3)}_{C_S} < \Gamma^{Q,(2)}_{C_S} - \Gamma^{P,(2)}_{C_S}$ is equivalent to

$$\frac{\varphi}{\sigma+\varphi}\left(\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{(1-\alpha)(1+\alpha(\sigma-1))}\right) < \frac{\varphi}{1+\alpha\theta(\sigma+\varphi)}.$$

Straightforward algebra shows that this is equivalent to $\sigma + \varphi < (\sigma-1)/\theta$. The inequality in the other direction follows analogously. This concludes the proof of (18) in the proposition, which concludes the proof of Proposition 4. □

E Indeterminacy – closed-form results for a special case

This appendix complements the results in Section 3 of the paper. The proposition below derives in closed form (but under further assumptions on parameters) conditions under which the Taylor principle may fail to induce determinacy, that is under which determinacy requires a stronger response to inflation. It also shows that under the conditions given here, the Taylor principle can fail in the fixed-supply regime but not in the fixed-price regime.

The additional assumptions are that all of the E-good is owned by Foreign ($\iota = 0$) and that there is no heterogeneity in Home ($\nu = \lambda$, or $\lambda = 0$). For this case, one can show by paper and pencil that indeterminacy can arise in the case of fixed supply. Before stating the proposition, let us anticipate the results. The proposition shows that the Taylor principle is "the more likely" to fail to ensure determinacy (that a too weak response to inflation may fail to ensure determinacy even if it respects $\phi_\Pi > 1$) if one or more of the following is the case: if the Phillips curve absent the supply constraint is flat (ϵ/ψ low), if households are sufficiently unwilling to substitute intertemporally ($1/\sigma$ small), if the E-good is a sufficiently important input in production (α high), if the labor supply elasticity is sufficiently low (φ high), or if labor and the E-good are hard to substitute in production (θ small), or both.

Proposition E.1. *Consider the same conditions as in Proposition 1 in the main text. In addition, let the E-good be owned entirely by Foreign, $\iota = 0$. Abstract from household heterogeneity ($\nu = \lambda$).*[42] *Note that these are the same assumptions as in case (1) of Proposition 4 in the main text. Then the following is true:*

*1. In the **fixed-price regime** for the E-good we have that both $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} > 0$, so that any response $\phi_\Pi > 1$ ensures determinacy.*

*2. In the **fixed-supply regime** for the E-good the following statements are true*

2.a) the convolute parameters governing the slopes of IS and Phillips curve are given by

$$\widetilde{\sigma} = \frac{\sigma}{1-\alpha}\frac{1-\alpha\,(1+\varphi+1/\theta)}{1+\alpha(\sigma-1)} \qquad \text{and} \qquad \widetilde{\kappa} = \frac{\varepsilon}{\psi}\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)}.$$

2.b) If the fundamental parameters are such that $\widetilde{\kappa} < 0$, then $\widetilde{\sigma} < 0$ as well. But $\widetilde{\sigma} < 0$

[42] This allows, of course, also for the case that $\lambda = 0$, in which case the (then non-existent) hand-to-mouth households would not receive profit income either.

does not imply $\widetilde{\kappa} < 0$*. In addition,* $\widetilde{\sigma} > 0$ *implies* $\widetilde{\kappa} > 0$*.*

2.c) Suppose that the fundamental parameters satisfy

$$\begin{array}{lll} \textit{if } 1-\alpha(1+\varphi) \leq 0 & \textit{then} & \alpha\frac{\sigma-1}{\varphi+\sigma} < \theta, \\ \textit{if } 1-\alpha(1+\varphi) > 0 & \textit{then} & \alpha\frac{\sigma-1}{\varphi+\sigma} < \theta < \frac{\alpha}{1-\alpha(1+\varphi)}. \end{array} \tag{E.1}$$

Then $\widetilde{\sigma} < 0$ *and* $\widetilde{\kappa} > 0$*. In addition, in this case, any* $\phi_\Pi > 1$ *ensures determinacy if and only if the following inequality holds:*

$$\frac{1}{2}\frac{\varepsilon/\psi}{\sigma}\frac{1-\alpha}{\alpha}\frac{\varphi+\sigma-\alpha/\theta(\sigma-1)}{1+\varphi+1/\theta-1/\alpha} \geq 1. \tag{E.2}$$

If the inequality in (E.2) *is violated, instead,* ϕ_Π *needs to be sufficiently greater than 1 to ensure determinacy.*

2.d) Consider the same conditions as in 2.c). In addition, assume that $\alpha = \theta$*, meaning the weight of the E-good in production equals the elasticity of substitution between the E-good and labor. Then condition* (E.2) *simplifies to*

$$\frac{1}{2}\frac{\varepsilon/\psi}{\sigma}\frac{1-\alpha}{\alpha} \geq 1.$$

Proof. For all the following, recall that from Proposition 1 in the main text what matters for determinacy are the values of $\widetilde{\kappa}$ and $\widetilde{\sigma}$.

Proof of item 1 of Proposition E.1: By item 1 of Proposition 1 in the main text, the Taylor principle will ensure determinacy, whenever $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} > 0$. Here we show that in the fixed-price regime this will be the case, indeed. The sign of $\widetilde{\kappa}$ depends on the sign of Γ^P_Λ only. And $\Gamma^P_\Lambda > 0$ by (D.3) (the equation is in Appendix D.2). Similarly, the sign of $\widetilde{\sigma}$ depends only on $\Gamma^P_{C_S}$. For $\nu = \lambda$, this sign is positive by item 1 of Proposition 4 ((17) in the main text). This proves item 1 of the current proposition: the Taylor principle is alive and well in the fixed-price regime.

Proof of item 2 of Proposition E.1: This focuses on the fixed-supply regime throughout.

First focus on the expressions in 2a): $\widetilde{\kappa} = \epsilon/\psi\,\Gamma^Q_\Lambda$. By (D.2) (the equation is in Appendix D.2) this gives

$$\widetilde{\kappa} = \frac{\epsilon}{\psi}\frac{\sigma+\varphi-\frac{\alpha}{\theta}(\sigma-1)}{1+\alpha(\sigma-1)}, \tag{E.3}$$

which is identical to the expression in 2a). Next, $\widetilde{\sigma} = \sigma\Gamma^Q_{C_S}$. For this, combining (D.4) and (D.5) (both are in Appendix D.4), we have that

$$\widetilde{\sigma} = \frac{\sigma}{(1-\alpha)} \frac{1-\alpha\,[1+\varphi+1/\theta]}{1+\alpha(\sigma-1)}, \tag{E.4}$$

which again is identical to the expression in 2a). This proves statement 2a).

Next, focus on statement 2b): For this observe that (E.3) gives that

$$\widetilde{\kappa} < 0 \qquad \Longleftrightarrow \qquad 1-\frac{\alpha}{\theta} < -\frac{1}{\sigma}\left(\varphi+\frac{\alpha}{\theta}\right), \tag{E.5}$$

Similarly,

$$\widetilde{\sigma} < 0 \qquad \Longleftrightarrow \qquad 1-\frac{\alpha}{\theta} < \alpha(1+\varphi), \tag{E.6}$$

The left-hand sides of (E.5) and (E.6) are identical. As all the parameters are positive, the right-hand side in (E.5) is negative, however, whereas the right-hand side in (E.6) is positive. This proves statement 2b).

Next, focus on statement 2c): That these restrictions on parameters imply $\widetilde{\kappa} > 0$ and $\widetilde{\sigma} < 0$ follows directly from (E.5) and (E.6). By item 2. of Proposition 1 in the main text, determinacy then requires $\phi_\Pi > \max(1, -4\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1)$. For any $\phi_\Pi > 1$ to ensure determinacy, thus, we need $-4\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1 < 1$. Plugging in the expressions for $\widetilde{\sigma}$ and $\widetilde{\kappa}$ from (E.4) and (E.3) above, this boils down to (E.2) (bearing in mind that, here, $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$ by assumption). This proves the statement 2c).

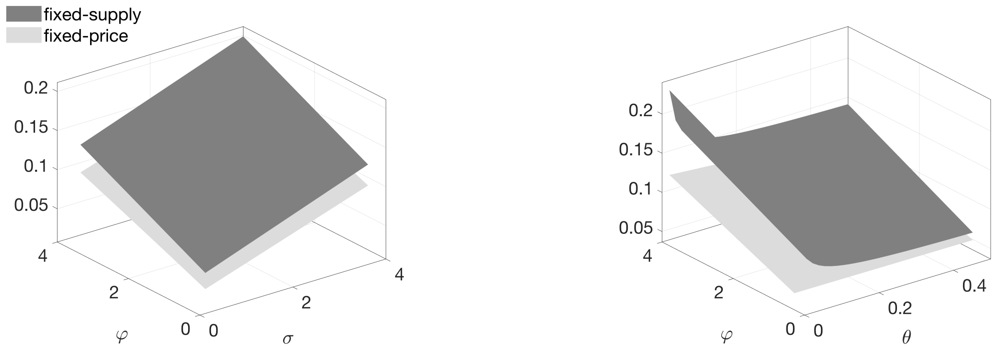
Last, focus on statement 2d): This follows directly from (E.2) for $\alpha = \theta$. □

F Slope of the Phillips curve graphically

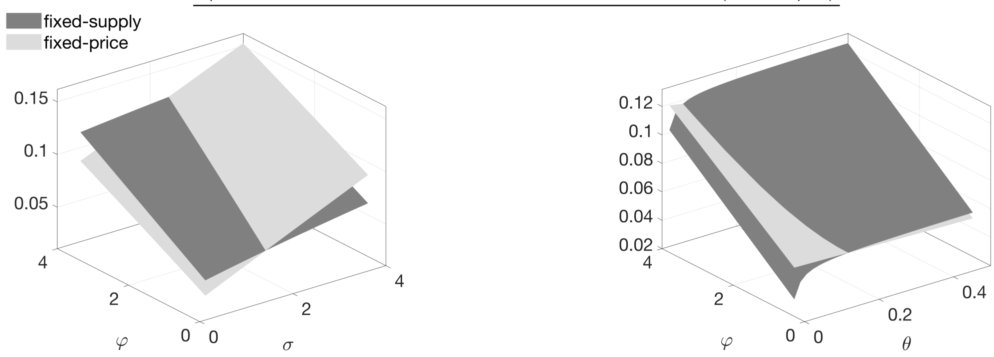
This appendix reports on the slope of the Phillips curve, $\widetilde{\kappa}$, and how that is affected by parameters. The slopes are reported for the fixed-price scenario and the fixed-supply scenario. The simplifying assumptions of Section 3 apply.

Figure F.1 Slope of the Phillips curve in production

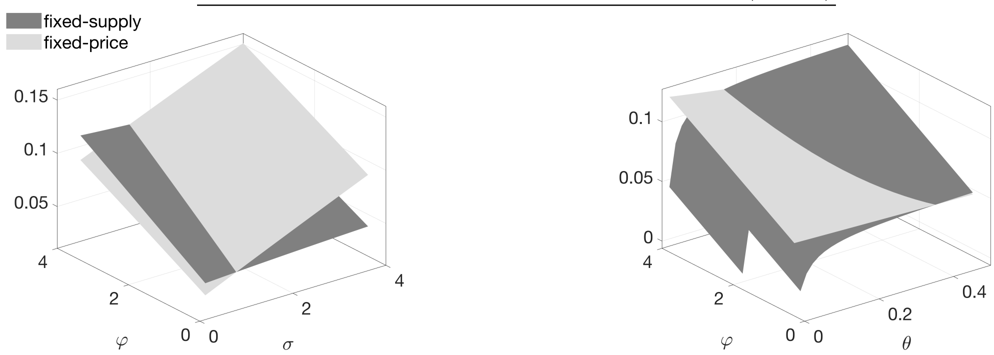
All energy owned domestically ($i = 1$)



1/3 of the energy owned domestically ($\iota = 1/3$)



None of the energy owned domestically ($\iota = 0$)



Notes: Slope of the Phillips curve, $\widetilde{\kappa}$. Light gray denotes fixed-price, dark fixed-supply regime for different ownership structures of energy (ι). The figures work under the assumptions spelled out in Section 3.1. Unless varied in the figure, the parameters are $\varepsilon = 11$, $\psi = 507$, $\theta = 0.1$, $\alpha = 0.077$, $\sigma = 2$, $\varphi = 3$.

G Policy options to avoid the feedback loop

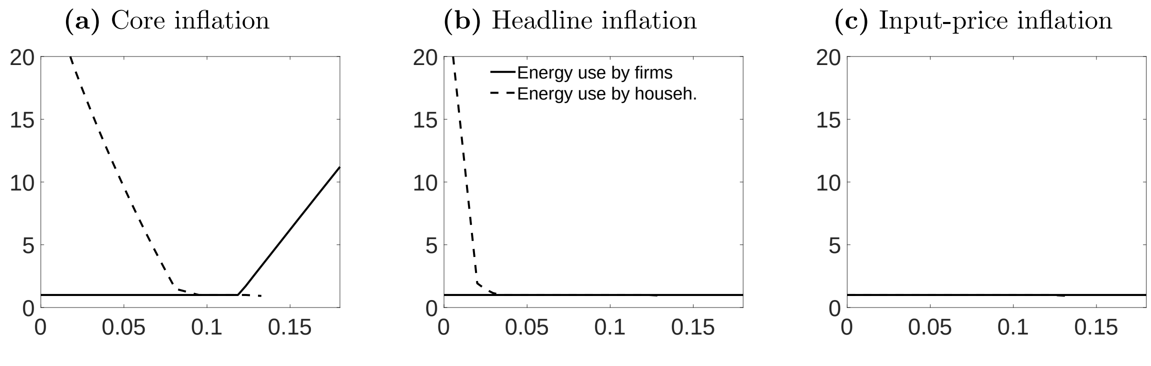
Section 4.5 in the main text illustrates the scope for indeterminacy in a crisis scenario. The section also briefly discusses policy options to avoid the feedback loop. The current appendix discusses the policy options in greater detail.

G.1 Domestic monetary policy

What all monetary policy options that help prevent the feedback loop have in common is that the central bank needs to be more hawkish in the crisis scenario than in normal times. The Taylor principle, in particular, can no longer be taken for granted. For example, with a response to core inflation, only a response of $\phi_\Pi > 8.23$ prevents the feedback loop.

Measures of inflation that do not "look through" higher energy prices. What underpins the feedback loop in Section 4.5 of the main text is a redistribution from savers to both hand-to-mouth households and Foreign. What is key to this is that wages and energy prices rise when aggregate demand rises. A central bank that, directly or indirectly, leans against such price increases thus also leans against the feedback loop. For example, in the parameterization that underpins the crisis scenario, a response to the change in nominal marginal costs (that is, to input price inflation) and a response to headline consumer price inflation come with a conventional determinacy cutoff of $\phi_\Pi > 1$. Both of these measures of inflation put some weight on the energy price.
Figure G.1 shows how the determinacy cutoffs of such policies depend on the shares of energy in consumption and production. For any value of ϕ_Π (the monetary response

Figure G.1 Energy intensity and the determinacy cutoff



Energy intensity households/firms

Notes: The figure plots the determinacy cutoff (y-axis), varying the steady-state share of energy expenditures of households or firms (x-axis). From left to right: core inflation in the Taylor rule, headline inflation, and input price inflation. Solid line: the expenditures of firms vary, while expenditures of households remain at the crisis value of 7.788 percent of GDP. Dashed line: the expenditures of households vary, keeping the expenditures of firms at the steady-state share of 16.212 percent of GDP. Only parameters α, γ, and $\bar{e}$ (the latter to keep the subsistence share constant) vary. All other parameters remain fixed at their values in the "crisis" parameterization, compare Table 1 in the main text.

to inflation) larger than the cutoff (above the respective lines), the equilibrium will be unique. For any value lower than that, the feedback loop arises. Panel (a) refers to a response to core inflation. Here, the larger the share of energy used by firms (the solid line) rather than households (the dashed line), the higher the determinacy cutoff. Thus, the feedback loop crucially depends on the use of energy in production.[43] Panel (b) refers to a Taylor rule that has interest rates respond to headline inflation. The dashed line once more varies steady-state energy use by households, keeping the energy use by firms constant. In this case, as long as households' energy consumption share is sufficiently large relative to firms' production share (that is, γ/α is sufficiently large), a monetary response to headline inflation that respects the Taylor principle ensures determinacy.[44] Panel (c) shows the case of a response to input price inflation. Responding to input-price inflation is fail-safe in that adhering to the Taylor principle ensures determinacy independent of the weight of energy in production or consumption.

Response to economic activity. In the feedback loop higher energy prices go hand in hand with higher employment or *vice versa*. A central bank that tightens when employment is high (unemployment is low) thus leans against the feedback loop. Quantitatively, consider a central bank that responds to core inflation with the baseline weight of $\phi_\Pi = 1.5$. A response to employment with $\phi_N > 0.39$ restores determinacy. A response to GDP, however, would further exacerbate the feedback loop. The reason is that the feedback loop comes with higher inflation and lower GDP. Leaning against the fall in GDP further fuels the feedback loop.[45]

G.2 Domestic fiscal policy

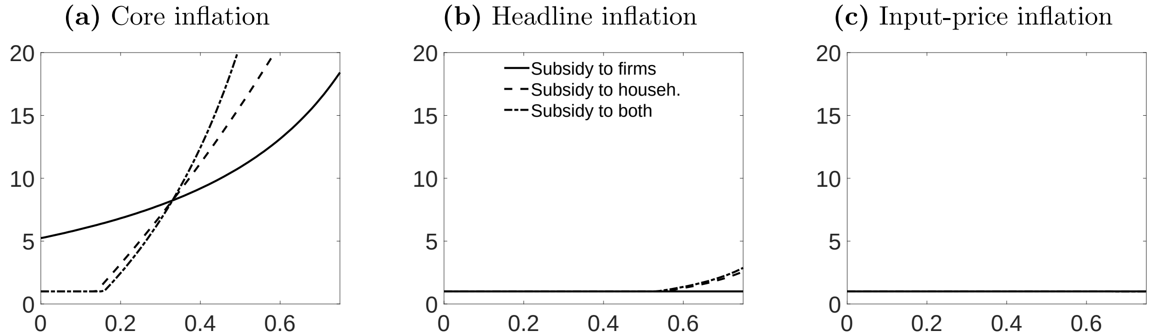
In the "crisis" scenario that we explored in Section 4.5 of the main text, a fiscal policy intended to shield households and firms from higher energy prices serves to support the wholesale price of energy as part of a demand-driven feedback loop. This in turn supports foreign demand, labor demand, and wages. Figure G.2 looks into which domestic fiscal-monetary mix prevents the feedback loop. The three panels refer to the same three monetary policies discussed in Figure G.1. As before, the y-axis shows the determinacy cutoff. On the x-axis the figure varies the subsidies for households, τ_E^c, or for firms, τ_E^f, or both. Focus on Panel (a) first. The dashed line varies the subsidy to households (τ_E^c, x-axis) but keeps the subsidy to firms (τ_E^f) at the value of 0.33, which is used in the crisis scenario. Under the parameterization of the "crisis" scenario, if the subsidy to households remains below 15 percent of the price, the Taylor principle holds, and there

[43]This is why Section 3 in the main text has focused on the use of the E-good in production only.

[44]For a given value of γ, if α increases sufficiently, the same result emerges (not shown in the figure).

[45]In the parameterization here, the rules commonly known as Taylor-1993 and Taylor-1999, which have coefficients on GDP of 0.125 and 0.25, respectively, induce determinacy if employed with headline inflation, and indeterminacy if employed with core inflation.

Figure G.2 Energy subsidies and the determinacy cutoff



Notes: The figure plots the determinacy cutoff (y-axis), varying energy price subsidies – either τ_E^c or τ_E^f or both at the same time (x-axis). From left to right: core inflation in the Taylor rule, headline inflation, and input-price inflation. Solid line: the subsidy to firms varies, while the one to households remains at the crisis value of 0.33. Dashed line: the subsidy to households varies, while the one to firms remains at the crisis value of 0.33. Dash-dotted line: both subsidies are varied. Only parameters τ_E^c and τ_E^f vary. All other parameters remain fixed at their values in the "crisis" parameterization; compare Table 1.

is no feedback loop. Beyond that, the determinacy cutoff rises steeply (the y-axis). In contrast, the subsidy to firms is of lesser importance. The solid line varies the subsidy to firms, keeping a subsidy for households of 0.33. Even absent subsidies on the production side (the left end of the solid line), the central bank would still need to respond strongly to inflation to ensure determinacy ($\phi_\Pi > 5.24$). With subsidies, households and firms are less inclined to substitute away from energy. This keeps the energy price elastic to output and strengthens the redistribution of incomes to both hand-to-mouth households and Foreign. On top of this, a subsidy to firms dampens the rise in marginal costs. Firms shift to labor to a lesser extent, thereby dampening the rise in wages and the domestic redistribution that underpins the feedback loop. The dashed-dotted line shows the effect of varying the two subsidies at the same time. The remaining two panels (Panels (b) and (c)) show that the stability implications of a Taylor rule targeting headline inflation or input-price inflation, respectively, are rather robust to the extent of subsidies, too.[46]

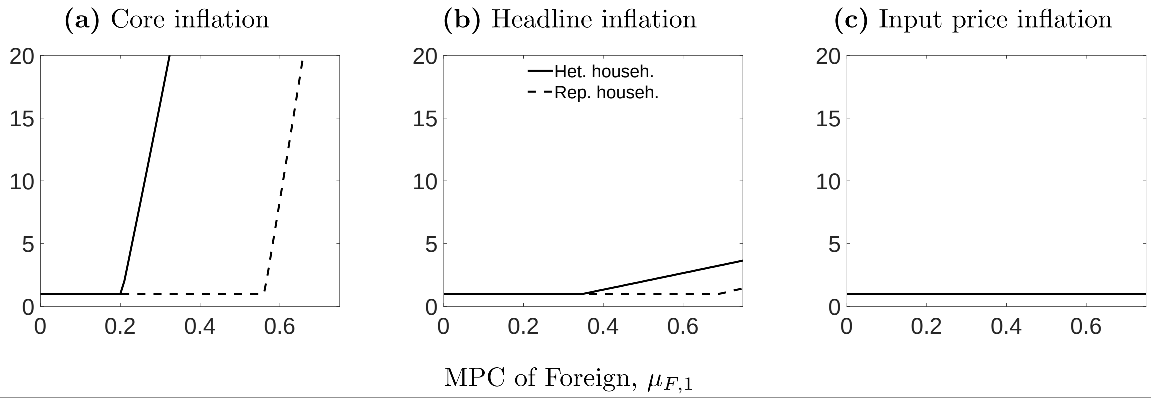
G.3 Foreign's monetary or fiscal policy

The analysis in the main text has parameterized Foreign's MPC out of windfall revenues from energy (parameter $\mu_{F,1}$). We chose a parameter that is in line with historical precedent. This MPC, however, is under (at least) the partial control of foreign monetary and fiscal policy. Because international coordination of such policies could also be a possibil-

[46]The modeling in the paper is, of course, silent about the effect of other government interventions such as working toward a more flexible or cheaper supply of energy in the first place or working toward storage and buffer stocks. All of these options, presumably, involve investment in infrastructure, which again would be relevant for demand, especially at shorter horizons–following, for example, the lines of Bilbiie, Känzig and Surico (2022).

ity, in the last section of this appendix here we report how Foreign's MPC shapes the constraints on domestic monetary policy. Toward this end, the three panels of Figure G.3

Figure G.3 Marginal propensities to consume and the determinacy cutoff



Notes: The figure plots the determinacy cutoff (y-axis) varying Foreign's MPC, $\mu_{F,1}$ (x-axis). From left to right: core inflation in the Taylor rule, headline inflation, and input price inflation. Solid line: "crisis" parameterization. Dashed line: representative household in Home, $\nu = \vartheta = \lambda$. All other parameters remain fixed at their values in the "crisis" parameterization, compare Table 1 in the main text.

again look at the same three different types of monetary policy. In each panel, the x-axis now varies Foreign's MPC out of energy revenue, $\mu_{F,1}$. For completeness, the figure not only reports the determinacy cutoffs for the model at hand but also for a counterfactual that abstracts from heterogeneity across households (the dashed lines). The indeterminacy can arise here, too, but the foreign MPC would need to be notably larger. The solid lines instead show the determinacy cutoff in the model at hand. Responding to input price inflation remains fail-safe in that the Taylor principle remains valid regardless of what the foreign MPC is (Panel (c)). For a response to headline inflation, obeying the Taylor principle ensures determinacy as long as the foreign MPC is around 0.35 or lower (Panel (b)). For a response to core inflation, the cutoff for the MPC is lower. Above a value of the foreign MPC of 0.21, the determinacy cutoff rises quickly.[47] An appropriate choice of domestic monetary policy obviates the need for coordination.

[47]Remember that the crisis parameterization has a value of $\mu_{F,1} = 0.25$.