Limited (energy) supply, sunspots, and monetary policy*

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Abstract

Recent changes in the macroeconomic environment have raised and likely will continue to raise the spectre of local shortages of production inputs. Limits to the supply of such goods mean that the local price of the goods may be rather elastic to domestic economic activity. In a simple open-economy New Keynesian setting, the paper explores conditions under which such shortages can raise the risk of self-fulfilling fluctuations. The paper highlights the role of ownership of the constrained factor, marginal propensities to consume, the factor's use in consumption and production, and fiscal policy. A firmer focus of the central bank on input prices (or on headline consumer prices) removes such risks.

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1 Introduction

The international macroeconomic environment increasingly appears to be affected by shortages of inputs. The underlying causes of such shortages can be manifold, ranging from supply chain disruptions driven by natural disasters to, for example, local or geopolitical policy decisions. What they have in common is a potentially profound change in the sensitivity of prices to local demand conditions. Namely, even from the perspective of a small open economy and for imported inputs, supply would no longer be abundant at a given price. With the local price feeding back to local demand conditions, such shortages can have potentially profound implications for monetary stabilization policy.

This paper studies one dimension of such shortages. Namely, it provides conditions under which self-fulfilling fluctuations can emerge from supply constraints on an input to consumption or production. Self-fulfilling fluctuations can arise when the supply constraint implies that high prices induce a redistribution of incomes from agents with a low marginal propensity to consume (MPC, hereafter) to agents with a high MPC. We emphasize the role of ownership of the constrained input, the extent to which the input is used in consumption and production, and of the fiscal response. Finally, provided that there is scope for self-fulfilling fluctuations, we ask how monetary policy could prevent them.

The theory we develop applies to a generic supply-constrained factor. For the remainder of this paper we will refer to this factor as "energy." We do this for ease of exposition and because we apply the theory to the European energy crisis following the Russian invasion of Ukraine.[1] We look at an open economy where goods are produced using labor and energy. Energy can be owned domestically or by the rest of the world. Both goods and energy are in the consumption baskets of households. Goods prices are rigid. Energy prices are flexible. This core of the model is the same as in Blanchard and Galí (2009). International trade does not have to balance immediately. To this, we add liquidity constraints as in Bilbiie (2021). Namely, a fraction of households receives labor

[1]The reader may also consider other inputs such as food crops, metals, or water. In the case of metals, for example, China just recently curtailed exports in July 2023. The reader might also think about "greenflation," that is the supply of green energy not keeping up with energy demand. For example, constraining the number of available emission certificates limits the supply of energy.

and transfer income only. These households consume all of their income. The rest of the households are savers. In addition to labor income, they receive dividends from firms and may own a share of the scarce factor. To reflect shortages, and contrary to the literature, energy is not abundantly available at an exogenous *price*, or elastically supplied. Rather, we treat the *quantity* of energy available to households and firms as fixed and the price as endogenous. We provide analytical intuition for the considerations under which energy scarcity exposes the economy to the risk of self-fulfilling fluctuations. We then explore such a scenario quantitatively by calibrating the model to reflect energy shortages in Germany following the Russian invasion of Ukraine.[2]

Supply constraints mean that a situation of self-fulfilling fluctuations could arise from a feedback loop between energy prices, economic activity, and the distribution of income. Namely, suppose that households and firms have a non-fundamental belief that energy prices will be high. If energy is imported, higher energy prices imply higher external demand, the strength of which depends on the propensity of the trading partner to demand exports when its energy-related income rises. In addition, under this belief, firms face high marginal costs. Rigid goods prices mean that firms cannot pass these costs on to consumers. Aggregate demand (domestic plus external) can, therefore, be high. Since the supply of energy is fixed, high demand can only be met by increasing the supply of labor, which requires higher wages. Higher wages mean higher incomes for domestic households with high MPCs. These support aggregate demand even when energy prices are high, validating the non-fundamental belief. In this feedback loop, high energy prices are a symptom of high demand. To rule out the loop, in an environment of high energy prices, the central bank would have to reduce domestic demand sufficiently much to reduce aggregate demand. We find that this may require a much stronger response to core inflation than is conventional, a response to headline inflation, or a response to input prices.

The gist of the policy implications is that, to break the energy-price-activity feedback loop, monetary policy must lean sufficiently strongly against rising input costs. It can

[2]While energy prices, and especially natural gas prices, have fallen from their 2022 peaks, the energy crisis does not appear to be resolved; see the International Energy Agency's July 2023 *Global Gas Security Review* (IEA, 2023b). Parts of Europe remain dependent on natural gas, much of it being delivered by Russia. The IEA, like us, emphasizes the role of supply flexibility—or rather, inflexibility.

do this directly (*raising* interest rates when energy prices *rise* or when *nominal wages* rise) or it can do so indirectly through its response to inflation and economic activity. A focus on headline inflation is more conducive to breaking the feedback loop than a monetary policy response focused on core inflation. The reason is that core inflation reflects the rise in energy prices to a lesser extent. Trying to stabilize real activity is risky because the feedback loop involves conflicting developments on the real side: rising output and employment, but falling value added (GDP). A monetary policy response aimed at stabilizing GDP would, thus, further fuel the feedback loop. Instead, a monetary response that dampens employment works against the loop.

We further examine the determinants of the feedback loop and the risks to macroeconomic stability. Several results emerge that we consider important. First, fiscal policy plays an important role.[3] Fiscal policies that provide energy price subsidies and shift the income gains associated with non-fundamental increases in energy prices to hand-to-mouth households further support the feedback loop. The reason is simple. To the extent that energy is used for consumption, a higher energy price on its own acts as a tax on income, dampening the demand for energy and goods, and, thus, easing the pressure on energy prices. Subsidies work in the opposite direction. Second, because of the dampening effect of higher prices on consumption demand, the more of the energy is consumed directly rather than used as an input in production, the less likely the feedback loop is to occur, all else being equal. Moreover, households' substitution out of energy implicitly makes the supply of energy to firms more elastic, again dampening the feedback loop.

The rest of the paper is structured as follows. We review the literature next. Section 2 presents the model. Section 3 provides pencil-and-paper results for special cases of the model, so as to provide intuition for the possibility that the feedback loop arises. Section 4 calibrates the model economy to a stylized German economy, analyzes the determinants of the feedback loop, and provides policy options. A final section concludes.

[3] Fiscal policy is important: The International Energy Agency estimates that in 2022 global fossil fuel subsidies skyrocketed to, by far, the highest levels ever seen (IEA, 2023a).

Related literature Our paper emphasizes that supply constraints can make it significantly more difficult for the central bank to anchor inflation expectations and economic activity. We consider an input factor that is fixed over the business cycle and study the implications for non-fundamental fluctuations. In this framework, the price of the fixed factor is endogenous and strongly influenced by the demand for it. We are not the first to consider the implications of this. Lucas and Prescott (1974) consider a framework in which the supply of labor is constrained and wages may rise steeply. Álvarez-Lois (2006), Fagnart, Licandro and Portier (1999) and Kuhn and George (2019) analyze the role of capacity constraints in the propagation of aggregate fundamental shocks. We do not consider fundamental fluctuations, but rather identify when fixed factors can lead to non-fundamental fluctuations. Boehm and Pandalai-Nayar (2022) provide empirical evidence for sizable convexities in the supply curves of US industries due to capacity constraints. One way to think about our framework is that we consider an economy where the capacity constraint is binding in the current business cycle. Balleer and Noeller (2023) argue that input constraints are empirically important for the transmission of monetary policy. Comin, Johnson and Jones (2023) analyze the role of occasionally binding capacity constraints (both, due to domestic factors and imported intermediates) for inflation working through supply chains. We, instead, focus on the distributional effects of factor scarcity and the implications for aggregate demand. Finally, Lorenzoni and Werning (2023) study wage-price spirals in a New Keynesian model with a fixed input factor and representative households.

The quantitative exercise interprets the supply-constrained factor as energy. There is, of course, a vast literature on energy and the macro-economy, to which we cannot do full justice here. Closest in terms of modeling are Blanchard and Galí (2009). They and a related paper, Blanchard and Riggi (2013), point to the structural features that shape the response to fundamental energy supply shocks; namely, the share of energy in production and consumption, the monetary response, and the degree of real wage rigidity. Nakov and Pescatori (2009) focus on the energy elasticity of output. Olivi, Sterk and Xhani (2022) and Känzig (2022) have analyzed the distributional effects of energy price changes.

All of these papers consider an environment of abundant energy supply, which precludes the energy-price-activity feedback loop that we study. Other papers, like ours, work with an exogenous energy supply, for example, Datta et al. (2021). Differences in the calibration explain why a feedback loop does not emerge in their work. Our calibration is based on Bachmann et al. (2022) who estimate the effect of an exogenous cut in Russian natural gas supply on the German economy, abstracting from nominal rigidities. Pieroni (2023) provides an assessment of a European energy stress scenario in a heterogeneous household New Keynesian model. Kharroubi and Smets (2023) study optimal fiscal policy when there are cuts in energy supply in a two-household setting with flexible prices. What distinguishes us from all these papers is that we study how restrictions on energy supply (or a fixed input factor) can lead to *self-fulfilling* energy-price-activity loops.

In our calibrated model, a 20 percent increase in energy prices is associated with a decline in GDP of about one percent. This is broadly in line with empirical estimates in the literature; for example, the effect of inventory demand shocks on global activity in Baumeister and Hamilton (2019), the SVAR-based findings in Blanchard and Galí (2009) and Blanchard and Riggi (2013), and the oil supply news shocks identified by Känzig (2021). This literature understands fluctuations to arise from exogenous fundamental shocks rather than the sunspots that drive prices in our environment.

We propose a novel mechanism that can generate an energy-price-activity feedback loop. This distinguishes the current paper from other papers that also challenge the Taylor principle. Bilbiie (2008) and Galí, López-Salido and Vallés (2004) derive the failure of the Taylor principle in a closed economy with limited participation in asset markets, as we do.[4] Our paper shares with the above what is technically at the heart of the indeterminacy: an inversion of the IS curve, that is, the relationship between aggregate output and the *ex-ante* real interest rate.[5]

Our results seem to contradict the conventional wisdom about the monetary response

[4] An earlier version of our paper included precautionary saving motives as in Bilbiie (2021). This was not essential to understanding the core mechanism, so we decided to simplify the exposition.

[5] Branch and McGough (2009) and Ilabaca and Milani (2021) derive a breakdown of the Taylor principle from adaptive expectations, Ascari and Ropele (2009) from trend inflation, Llosa and Tuesta (2009) from the cost channel of monetary policy, and Kurozumi (2006) from non-separable preferences.

to energy price shocks. In positive contributions, Carlstrom, Fuerst and Ghironi (2006) show that, in their setting, a central bank that responds more than one-to-one to the rate of inflation of *any* arbitrary subset of goods in the economy ensures determinacy; see also Airaudo and Zanna (2012). In our setting, it matters instead which price index the central bank targets. The reason is that price changes shape domestic and external demand. Responding to energy price changes, as would help ensure determinacy in our setting, seems to run counter to the normative implications of a long stream of literature, which finds that central banks should best focus on the inflation rate of those goods and services that have rigid prices rather than of those goods or services that have flexible prices. For the closed economy Aoki (2001) formalizes the notion that policy should react to inflation in goods that have rigid prices. Bodenstein, Erceg and Guerrieri (2008) do so for the open economy with an energy supply shock. Contributions for multi-sector models, such as Eusepi, Hobijn and Tambalotti (2011) and Rubbo (2022) come to similar conclusions. Such policies help to prevent non-fundamental fluctuations if they are implemented rigorously. Otherwise, we find that an additional response to energy prices can achieve such anchoring of beliefs.

2 Model

We look at an infinite-horizon model. Time t is discrete and marked by $t = 0, 1, \ldots$. One factor of production (labelled "energy") is in fixed supply. Its supply is denoted by ξ_E. There are two countries, Home and Foreign. The Home economy imports a share of its energy from a generic energy-exporting country (Foreign) in exchange for goods that are produced domestically. The remaining share of energy is owned domestically. International trade need not be balanced immediately. Rather, Foreign can accumulate net foreign assets in the form of domestic-currency bonds. Energy is used in two ways: it is consumed by households directly and it serves as an input factor for the production of consumption goods. Energy is not storable. This setting is as in Blanchard and Galí (2009). There are two types of households in Home: savers and spenders. Savers seek to smooth consumption over time. Their consumption demand is affected by their permanent income. Spenders, instead, have an MPC of unity. This setting on the household side follows Bilbiie (2008).[6]

2.1 Households

There is a unit mass of infinitely-lived households with identical preferences. Households split into two types. There is a representative hand-to-mouth household and a representative saver household, marked by subscripts H and S, respectively. Hand-to-mouth households cannot participate in financial markets. They consume all of their income. Savers, instead, optimize intertemporally. They can save in liquid, risk-free nominal bonds, the rate of return on which the central bank controls. The bonds are traded among savers and with Foreign. The mass of hand-to-mouth households is given by λ.

A household i, with $i \in \{H, S\}$, consumes a basket of goods, $C_{i,t}$. The household works and, potentially, saves. The household maximizes expected life-time utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right] \right\},$$

with parameters $\beta \in (0,1)$, $\sigma > 0$, $\chi > 0$, and $\varphi \geq 0$. $\mathbb{E}_t$ marks expectations conditional on period-t information. $N_{i,t}$ marks hours worked.

The consumption basket is comprised of the consumption of raw energy, $C_{i,E,t}$, and of non-energy goods, $C_{i,G,t}$. The latter are produced in Home. Consumption preferences are described by

$$C_{i,t} = \left[\gamma^{\frac{1}{\eta}} \left(C_{i,E,t} - \bar{e} \right)^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{i,G,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

Above, $\bar{e} \geq 0$ marks the subsistence level for the consumption of energy. $\gamma \in (0,1)$ measures the weight of energy in the consumption basket and $\eta > 0$ measures the household's elasticity of substitution between energy and goods. Marking the respective prices by $P_{E,t}^c$

[6] We abstract from transitions between types. An earlier version of this paper had such transitions, interpretable as "idiosyncratic risk;" following Bilbiie (2021). Such risk provides additional implications for monetary policy but is not necessary for describing the feedback loop. Therefore, we dropped it from this version of the paper.

and $P_{G,t}$, the household's allocation of consumption obeys

$$C_{i,E,t} - \bar{e} = \gamma \left(\frac{P^c_{E,t}}{P_t} \right)^{-\eta} C_{i,t}, \text{ and } \quad C_{i,G,t} = (1-\gamma) \left(\frac{P_{G,t}}{P_t} \right)^{-\eta} C_{i,t}.$$

The subscript c on $P^c_{E,t}$ indicates that this is the energy price that *households* face. This does not necessarily coincide with the wholesale price or the price that firms pay, since we allow the government to provide energy-price subsidies (see below). P_t marks the price of a marginal unit of consumption of the basket (that is, beyond subsistence), with

$$P_t = \left[\gamma (P^c_{E,t})^{1-\eta} + (1-\gamma)(P_{G,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{1}$$

For a hand-to-mouth household ($i = H$), the budget constraint is given by

$$P^c_{E,t} C_{H,E,t} + P_{G,t} C_{H,G,t} = W_t N_{H,t} + P_t T_{H,t}. \tag{2}$$

This household spends its income on energy and non-energy consumption. Income is derived from supplying labor in the competitive labor market, W_t being the nominal wage, and from lump-sum net transfers $T_{H,t}$.

For a saver household ($i = S$), the budget constraint is given by

$$P^c_{E,t} C_{S,E,t} + P_{G,t} C_{S,G,t} + \frac{1}{1-\lambda} B_t = W_t N_{S,t} + P_t T_{S,t} + \frac{1}{1-\lambda} R_{t-1} B_{t-1}. \tag{3}$$

This reflects that savers can accumulate savings or liabilities against other savers or Foreign. B_t marks the nominal expenditure for the bonds that underpin these savings. On the income side, savers have labor earnings and they receive lump-sum net transfers. The final term in the budget constraint marks the income that accrues to the saver household from the bond position, R_t being the nominal interest rate. Firms are not traded. Instead, households are endowed with the cash-flows of the firms. To facilitate the expositions, we have these cash-flows run through the lump-sum transfers, $T_{S,t}$. See Section 2.3 for the fiscal block. The labor-supply decision of both types of household is given by

$$W_t/P_t = \chi C_{i,t}^{\sigma} N_{i,t}^{\varphi}. \tag{4}$$

Hand-to-mouth households do not participate in financial markets. Only savers, thus, have an intertemporal consumption decision. Their Euler equation is given by

$$C_{S,t}^{-\sigma} = \mathbb{E}_t \left[\beta C_{S,t+1}^{-\sigma} R_t/\Pi_{t+1}\right]. \tag{5}$$

2.2 Firms

The setup of production follows the usual New-Keynesian structure. There is a unit mass of producers of differentiated goods, indexed by $j \in [0,1]$, that are subject to price adjustment costs. Each differentiated good, $y_{G,t}(j)$, is produced using labor and energy. A representative competitive retailer buys the differentiated goods and assembles them into the consumption good that consumers then purchase at the competitive price $P_{G,t}$. The representative retailer transforms the differentiated inputs into the (non-energy) consumption good according to production function

$$Y_{G,t} = \left[\int_0^1 y_{G,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here $\varepsilon > 0$ is the elasticity of substitution between the different differentiated inputs. The retailer takes prices $P_{G,t}(j)$ of intermediate inputs and $P_{G,t}$ of output as given. Profit maximization leads to the conventional Dixit-Stiglitz demand function

$$y_{G,t}(j) = \left(\frac{P_{G,t}(j)}{P_{G,t}}\right)^{-\varepsilon} Y_{G,t}, \tag{6}$$

with $P_{G,t} = \left[\int_0^1 P_{G,t}(j)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}$ being the producer-price index.

A unit mass of firms produce differentiated goods. The firms are not traded and discount profits as would savers. The producer sets its price $P_{G,t}(j)$ so as to maximize

$$\mathbb{E}_t \Bigg\{ \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{S,t+k}}{C_{S,t}} \right)^{-\sigma} \frac{1}{P_{t+k}} \Bigg[P_{G,t+k}(j)(1+\tau^y) y_{G,t+k}(j) - W_{t+k} N_{t+k}(j) \\ - P^f_{E,t+k} E_{t+k}(j) - \frac{\psi}{2} P_{G,t+k} Y_{G,t+k} \left(\frac{P_{G,t+k}(j)}{P_{G,t+k-1}(j)} - 1 \right)^2 \Bigg] \Bigg\}.$$

Here, $\tau^y \geq 0$ is a constant subsidy to production and $\psi > 0$ indexes the price adjustment costs. N_t and E_t mark labor and energy input into production, respectively. As with the consumer side, here the subscript f on $P^f_{E,t}$ indicates that this is the energy price that *firms* pay. Maximization is subject to demand function (6) and the production function

$$y_{G,t}(j) = \left[\alpha E_t(j)^{\frac{\theta-1}{\theta}} + (1-\alpha) N_t(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

Here, $\alpha \in (0,1)$ marks the input share of energy in production and $\theta > 0$ marks the elasticity of substitution of energy and hours worked in production.

Symmetry means that all differentiated goods producers set the same price, produce the same amount of output, and face the same marginal costs. The first-order conditions imply the New Keynesian Phillips curve

$$\psi \Pi_{G,t}(\Pi_{G,t} - 1) = (1+\tau^y)(1-\varepsilon) + \varepsilon \Lambda_t \left(\frac{P_{G,t}}{P_t} \right)^{-1} \\ + \psi \mathbb{E}_t \left[\beta \left(\frac{C_{S,t+1}}{C_{S,t}} \right)^{-\sigma} \Pi_{G,t+1}(\Pi_{G,t+1} - 1) \frac{Y_{G,t+1}}{Y_{G,t}} \frac{P_{G,t+1}/P_{t+1}}{P_{G,t}/P_t} \right]. \tag{7}$$

Above, $\Pi_{G,t} = P_{G,t}/P_{G,t-1}$ is goods-price inflation. Note that in the current setting, this is the same as producer-price inflation and core inflation. Λ_t marks real marginal costs, real in terms of the consumption aggregate. The optimal factor input shares obey

$$\frac{W_t}{P^f_{E,t}} = \frac{1-\alpha}{\alpha} \left(\frac{E_t}{N_t} \right)^{1/\theta}. \tag{8}$$

Real marginal costs are given by

$$\Lambda_t = \left[\alpha^\theta \left(P^f_{E,t}/P_t\right)^{1-\theta} + (1-\alpha)^\theta \left(W_t/P_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{9}$$

Last, the firm sector's real profits, in equilibrium are given by

$$D_t = (1+\tau^y)\frac{P_{G,t}}{P_t}Y_{G,t} - \frac{W_t}{P_t}N_t - \frac{P^f_{E,t}}{P_t}E_t - \frac{\psi}{2}\frac{P_{G,t}}{P_t}Y_{G,t}\left(\Pi_{G,t}-1\right)^2.$$

2.3 Fiscal policy, ownership of energy and firms

The fiscal arm of the government raises fiscal revenue and provides energy-price subsidies. We also use the government budget constraint to simplify the exposition as regards to who owns the supply-constrained factor (energy) and the firms. Namely, we assume that revenues from both accrue to the government and that the government distributes the revenues to households. Since all the fiscal instruments other than the energy-price subsidies are lump-sum and the government does not issue debt, having the accounting this way rather then through the household budget constraints is without consequence.

2.3.1 Energy-price subsidies

Fiscal policy may subsidize the price of energy. Let $P_{E,t}$ mark the competitive wholesale price that the owners of energy receive in exchange for energy. We assume that fiscal policy ensures that the retail price of energy, that households pay, evolves according to

$$\log(P^c_{E,t}/P_t) - \log(P_E/P) = (1-\tau^c_E)\big[\log(P_{E,t}/P_t) - \log(P_E/P)\big].$$

That is, there are no subsidies in steady state, but $\tau^c_E \in [0,1]$ is the proportional energy subsidy for households when the price rises above the steady-state level. The case of $\tau^c_E = 0$ marks the case without subsidies: households pay the wholesale price. When $\tau^c_E = 1$, fiscal policy perfectly stabilizes the real retail price of energy at its steady-state value.[7] Likewise, fiscal policy can subsidize the price of energy for firms at

$$\log(P^f_{E,t}/P_t) - \log(P_E/P) = (1-\tau^f_E)\big[\log(P_{E,t}/P_t) - \log(P_E/P)\big],$$

where $\tau^f_E \in [0,1]$ marks the proportional subsidy for firms' energy use.

2.3.2 Transfers and the government budget constraint

The government runs a balanced budget and does not issue debt. Let $\iota \in [0,1]$ mark the share of the fixed energy endowment ξ_E that is owned by households in Home. Let $\vartheta \in [0,1]$ be the share of this endowment that, in turn, is owned by hand-to-mouth households. That is, hand-to-mouth households as a group have a claim to energy revenue of $\iota\vartheta\, P_{E,t}\xi_E$. Similarly, let $\nu \in [0,1]$ be the share of firms' dividends (net of the expenses for subsidizing production) that accrues to hand-to-mouth households.
In nominal terms, the government budget constraint, then, is given by

$$P_tD_t + \iota P_{E,t}\xi_E = \; (P_{E,t} - P^c_{E,t})C_{E,t} + (P_{E,t} - P^f_{E,t})E_t + \lambda P_t T_{H,t} + (1-\lambda)P_t T_{S,t} + \tau^y P_{G,t}Y_{G,t}.$$

Here, the left-hand side has the economy-wide dividends and the revenues from the energy endowment. The right-hand side has the expenses for the energy price subsidies in consumption and production, with $C_{E,t} := (\lambda\, C_{H,E,t} + (1-\lambda)\, C_{S,E,t})$ being aggregate energy consumption. Furthermore, the right-hand side features the lump-sum transfers to both households. The last term reflects the production subsidies. The transfers to hand-to-mouth households are given by

$$\lambda P_t T_{H,t} = \nu\big(P_tD_t - \tau^y P_{G,t}Y^G_t\big) + \iota\,\vartheta\, P_{E,t}\xi_E,$$

so that transfers reflect a claim to firms' profits (net of subsidies) and proceeds from a share of energy revenue. The transfers to savers, $T_{S,t}$, in turn, balance the budget. For the business cycle, since savers are Ricardian, we consider this the most innocuous assumption

[7] The model does not have steady states for nominal prices; thus, the scaling by P_t.

on government funding one can make.

2.4 Monetary policy

The central bank controls the gross nominal interest rate R_t. A common prescription for the optimal response of monetary policy to relative price changes is that the central bank should focus on the inflation rates of those goods or services that have rigid prices; see, for example, Aoki (2001). In the current context, this would mean to focus on the inflation rate associated with non-energy goods. Therefore, in the baseline, the Taylor rule has interest rates respond to "core inflation"[8]

$$R_t = R \cdot (\Pi_{G,t})^{\phi_\Pi}\,, \text{ where } \quad \phi_\Pi \geq 0. \tag{10}$$

2.5 International trade

A share $(1 - \iota)$ of the energy is supplied by Foreign. Foreign's budget equation (expressed in units of Home's currency) is, thus,

$$(1 - \iota)P_{E,t}\xi_E = P_{G,t}X_{G,t} - [B_t - R_{t-1}B_{t-1}]. \tag{11}$$

On the left-hand side, there are the revenues from the energy that Foreign sells into Home's market at the wholesale price $P_{E,t}$. The right-hand side shows that these revenues can be used for importing goods (Home's exports to Foreign, $X_{G,t}$) or for accumulating net foreign assets (nominal bonds issued in Home's currency).

We do not seek to model the Foreign economy in any more detail. Instead, we wish to control Foreign's MPC out of any windfall gains from an increasing energy price. Let $-b_t := -B_t/P_t$ mark Home's real obligations to Foreign. We parameterize Foreign's

[8] In Section 4.4, we will also consider a response of the central bank to other concepts of inflation. Recall that the "Taylor principle" asserts that $\phi_\Pi > 1$ would ensure a unique bounded equilibrium irrespective of what inflation index the central bank responds to.

demand for goods produced in Home as

$$\log(X_{G,t}/X_G) = \mu_{F,1}\log(Y_t^*/Y^*) - \mu_{F,2}\frac{b_{t-1}}{Y^*}.$$

Here, $Y_t^* = (1-\iota)\xi_E\, P_{E,t}/P_{G,t}$ marks current energy revenue of Foreign (expressed in units of Home's exports). Parameters $\mu_{F,1} \geq 0$ and $\mu_{F,2} \geq 0$ measure, respectively, Foreign's contemporaneous MPC out of energy revenues and its MPC out of savings.

2.6 Market clearing

In equilibrium, all markets clear. The labor market clears if $N_t = \lambda N_{H,t} + (1-\lambda)N_{S,t}$, that is, labor demand by firms equals the different households' labor supply. Next, the energy supply needs to be in line with the energy demand by households and firms, that is, $\xi_E = C_{E,t} + E_t$. Let $C_{G,t} = \lambda C_{H,G,t} + (1-\lambda)C_{S,G,t}$ mark total domestic consumption of non-energy goods. The goods market clears if

$$Y_{G,t} = C_{G,t} + X_{G,t} + \frac{\psi}{2}\left(\Pi_{G,t}-1\right)^2 Y_{G,t},$$

that is, if production equals demand for the good for consumption in Home plus export demand plus price adjustment costs.

2.7 Additional definitions: aggregates and inflation

For future use, we define further concepts of inflation and we define GDP. In the current setting, producer price inflation is the same as core consumer price inflation, the inflation rate being $\Pi_{G,t}$. A measure of headline consumer-price inflation is given by $\Pi_t := P_t/P_{t-1}$.[9] A measure of input price inflation is given by the change in nominal marginal costs as

$$\Pi_{\mathrm{nmc,t}} = \frac{\left[\alpha^\theta (P^y_{E,t})^{1-\theta} + (1-\alpha)^\theta (W_t)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[\alpha^\theta (P^y_{E,t-1})^{1-\theta} + (1-\alpha)^\theta (W_{t-1})^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \tag{12}$$

[9] This captures only the change in costs of marginal consumption (beyond subsistence). The results below are very similar when measuring consumer price inflation accounting for subsistence consumption.

compare equation (9). Last, nominal gross domestic product $P_t GDP_t$ is defined as

$$P_t \, \mathrm{GDP}_t = P_{G,t} C_{G,t} + P_{E,t} C_{E,t} + P_{G,t} X_{G,t} - (1-\iota) P_{E,t} \xi_E. \tag{13}$$

3 Pencil-and-paper intuition

This paper's main result is that limits to energy supply (or a fixed factor of production more generally) can give rise to a self-fulfilling feedback loop between energy prices and economic activity. This section provides pencil-and-paper intuition for this claim. First, we show the determinants of the feedback loop in a representative-household version of the model. Then, we discuss the interaction of the feedback loop with the heterogeneity of households. Throughout, we work with the linearized version of the model.

3.1 Parametric assumptions

This section focuses on energy as an input to production only. That is, we look at the case $\gamma \to 0$ and $\bar{e} \to 0$. This also means that core and headline consumer price inflation are identical, that is, $\Pi_{G,t} = \Pi_t$. We assume balanced trade, that is, $\mu_{F,1} = 1$. This makes sure that we do not need to keep track of the net foreign asset position. In turn, this allows us to describe the model by a system of just two equations (the IS equation and the Phillips curve) plus the Taylor rule. Production subsidies τ^y are such that the steady state is efficient. Energy prices are not subsidized, $\tau_E^f = 0$. All energy is imported from Foreign, $\iota = 0$. We make a few additional assumptions that are not essential, but simplify the exposition (the formulae that we show). Energy supply is fixed at $\xi_E = 1$. Next, we assume that the scaling parameter of the disutility of work, χ, is such that in the steady state the labor supply of all households equals unity. Last, we look at the limit $\beta \to 1$. We look at two versions of the model. Both of these share the assumptions above. In addition, the *representative-household* version of the model sets $\lambda = 0$ such that there are saver households only. The *heterogeneous-household* version, instead, allows for $\lambda > 0$. For this case, tractability requires further assumptions, however. Namely, in the heterogeneous-household version we set $\sigma = 1$ (no wealth effects on labor supply). In addition, we assume that there is a lump-sum scheme of constant taxes and transfers on households, T_H and T_S, that is designed such that it equalizes savers' and spenders' consumption in the steady state. That is, in steady state all households have the same disposable income irrespective of the extent to which spender households share in profit income or energy income, that is, of parameters ν and ϑ.

3.2 Representative household: Steady state and dynamics

We start with the representative household version. It is instructive to present the steady state and the first-order dynamics before presenting the key proposition.

Steady state Let variables without time index denote their steady state values. In steady state, inflation is given by $\Pi_G = 1$, the gross nominal interest is given by $R = 1/\beta$, and hours worked by $N = 1$. Production is given by $Y_G = 1$, marginal costs by $\Lambda = 1$, and consumption in Home is given by $C = (1 - \alpha)$. Let $q_t := P_{E,t}/P_t$ be the real price of energy and let $w_t := W_t/P_t$ be the real wage. In steady state, $q = \alpha$, and $w = (1 - \alpha)$. α, thus, marks the equilibrium share of energy in production.

Linearized equilibrium dynamics Let a hat mark percentage deviations of a variable from the steady state outlined above. The following system of seven equations in seven unknowns describes the evolution of the economy up to a first-order approximation around the steady state. The consumption Euler equation (after substituting the central bank's Taylor rule) gives $-\sigma\widehat{C}_t = -\sigma\mathbb{E}_t\widehat{C}_{t+1} + \left[\phi_\Pi\widehat{\Pi}_{G,t} - \mathbb{E}_t\widehat{\Pi}_{G,t+1}\right]$. The households' labor supply first-order condition gives $\widehat{w}_t = \varphi\widehat{N}_t + \sigma\widehat{C}_t$. The Phillips curve gives $\psi\widehat{\Pi}_{G,t} = \psi\beta\mathbb{E}_t\widehat{\Pi}_{G,t+1} + \varepsilon\widehat{\Lambda}_t$. The firms' first-order conditions for factor inputs give $\widehat{w}_t = \widehat{\Lambda}_t + \frac{1}{\theta}[\widehat{Y}_{G,t} - \widehat{N}_t]$, and $\widehat{q}_t = \widehat{\Lambda}_t + \frac{1}{\theta}\widehat{Y}_{G,t}$, where we have already used that energy is in fixed supply. Goods-market and energy-market clearing imply $\widehat{Y}_{G,t} = (1 - \alpha)\widehat{C}_t + \alpha\widehat{q}_t$. Last, the production function implies $\widehat{Y}_{G,t} = (1 - \alpha)\widehat{N}_t$.

Consolidating the IS equation and the goods-market clearing condition, we have

$$\widehat{Y}_{G,t} - \alpha\widehat{q}_t = \mathbb{E}_t\widehat{Y}_{G,t+1} - \alpha\mathbb{E}_t\widehat{q}_{t+1} - \frac{(1-\alpha)}{\sigma}\left[\phi_\Pi\widehat{\Pi}_{G,t} - \mathbb{E}_t\widehat{\Pi}_{G,t+1}\right].$$

Combining labor demand and supply as well as the goods-market clearing condition, one can further show that marginal costs are given by

$$\widehat{\Lambda}_t = \left[\frac{1}{1-\alpha}\left[\varphi+\sigma+1/\theta\right] - \frac{1}{\theta}\right]\widehat{Y}_{G,t} - \frac{\sigma\alpha}{1-\alpha}\widehat{q}_t.$$

The energy-demand equation of firms gives $\widehat{\Lambda}_t = -\frac{1}{\theta}\widehat{Y}_{G,t} + \widehat{q}_t$, so that the equilibrium price of energy is given by

$$\widehat{q}_t = \frac{\varphi+\sigma+1/\theta}{1-\alpha+\sigma\alpha}\widehat{Y}_{G,t}.$$

That is, the energy price is the more elastic to output, the less substitutable energy is as an input (the smaller θ) and the less elastic labor supply is (the larger φ). With this, real marginal costs are given by

$$\widehat{\Lambda}_t = \frac{\sigma+\varphi+\alpha/\theta(1-\sigma)}{1-\alpha+\sigma\alpha}\widehat{Y}_{G,t}.$$

Two-equation representation Combining all this, we can write the model as a system of two equations in output $\widehat{Y}_{G,t}$ and inflation $\widehat{\Pi}_{G,t}$. The IS-equation is given by

$$\widehat{Y}_{G,t} = \mathbb{E}_t\widehat{Y}_{G,t+1} - \frac{1}{\widetilde{\sigma}}\left[\phi_\Pi\widehat{\Pi}_{G,t} - \mathbb{E}_t\widehat{\Pi}_{G,t+1}\right], \tag{14}$$

with $\widetilde{\sigma} := \frac{\sigma}{1-\alpha}\frac{1-\alpha-\alpha\varphi-\alpha/\theta}{1-\alpha+\sigma\alpha}$.

That is, relative to the textbook New-Keynesian model, what changes is the interest-sensitivity of aggregate demand, $\widetilde{\sigma}$. If $\alpha[\varphi+1/\theta]$ becomes sufficiently large, the sensitivity changes sign. The Phillips curve in turn is given by

$$\widehat{\Pi}_{G,t} = \beta\mathbb{E}_t\widehat{\Pi}_{G,t+1} + \widetilde{\kappa}\,\widehat{Y}_{G,t}, \tag{15}$$

with $\widetilde{\kappa} := \frac{\varepsilon}{\psi}\frac{\sigma+\varphi+\alpha/\theta(1-\sigma)}{1-\alpha+\sigma\alpha}$.

3.3 Representative household: main pencil-and-paper results

The two equations (14) and (15) summarize the evolution of output and inflation. This means that the analysis of (in)determinacy can conceptually follow standard lines. This gives us the following proposition.

Proposition 1. *Consider the model of Section 2 and apply the assumptions listed in Section 3.1 for the representative-household case. Then two cases summarize the conditions for local determinacy:*

1) Conventional: If $\widetilde{\sigma}$ and $\widetilde{\kappa}$ have the same sign, determinacy holds if and only if $\phi_{\Pi} > 1$.

2) Potentially unconventional: If $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$, determinacy holds if and only if

$$\phi_{\Pi} > \max\left(1, -4\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right). \tag{16}$$

Proof. The proof is provided in Appendix A.1 and is entirely standard. It follows the lines of the proof of (in)determinacy for the textbook New Keynesian model, such as the one in Woodford (2003, p. 670 ff). $\widetilde{\kappa}$ can be negative only when $\widetilde{\sigma}$ is negative as well. This explains why there are two cases only and not a third case (no $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} < 0$). □

The proposition shows that obeying the standard Taylor principle ($\phi_{\Pi} > 1$) may not be sufficient to ensure determinacy. It will not be sufficient if $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$, and if $|\widetilde{\sigma}/\widetilde{\kappa}|$ is large. The following corollary clarifies the conditions under which this can be the case for an instructive special case. Appendix A.2 reports the general case.

Corollary 1. *Consider the same conditions as in Proposition 1. In addition, suppose that $\alpha = \theta$, meaning the weight of energy in production equals the elasticity of substitution between energy and labor. This implies that case 2) of the proposition is the relevant case (that is, $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$). We have the following result: An arbitrary response $\phi_{\Pi} > 1$ ensures determinacy if and only if the following inequality holds:*

$$\frac{1}{2}\,\frac{\varepsilon/\psi}{\sigma}\,\frac{1-\alpha}{\alpha} \geq 1.$$

If the above inequality is violated, determinacy requires a stronger response to inflation *than suggested by the Taylor principle.*

Proof. The proof is in Appendix A.2. The corollary follows directly from Proposition 1. If $\alpha = \theta$, we have that $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$—case 2) of the proposition. The result here emerges after substituting the expressions for $\widetilde{\sigma}$ and $\widetilde{\kappa}$ in inequality (16) (and using $\alpha = \theta$). $\square$

That is, whenever parameters are such that the inequality holds, the Taylor principle suffices to ensure determinacy. If not, there are values $\phi_\Pi > 1$ for which indeterminacy occurs and the feedback loop can arise. The inequality can be violated if the Phillips curve, absent energy-price feedback, is sufficiently flat (low ε/ψ), if households are sufficiently unwilling to substitute intertemporally (high σ), and if energy inputs are sufficiently important in production (high α).

3.4 Heterogeneous-household economy

Next, we focus on how a fixed factor ($\alpha > 0$) and household heterogeneity ($\lambda > 0$) together shape the possibility that the feedback loop emerges. For the case of the heterogeneous-household version of the model, we have the following proposition.

Proposition 2. *Consider the model of Section 2 and apply the assumptions listed in Section 3.1 for the heterogeneous-household case. Then, the economy can be represented by the two equations (14) and (15), where only the mapping from structural parameters into convolute parameters $\widetilde{\sigma}$ and $\widetilde{\kappa}$ changes relative to the representative-household case. Here $\widetilde{\kappa} = \frac{\varepsilon}{\psi}(1 + \varphi) > 0$. The conditions for determinacy (in terms of $\widetilde{\kappa}$ and $\widetilde{\sigma}$) are the same as in Proposition 1. Indeterminacy, thus, can occur only if $\widetilde{\sigma} < 0$. As regards the effect of fundamental parameters on $\widetilde{\sigma}$, we have that $\partial\widetilde{\sigma}/\partial\alpha < 0$ and $\partial^2\widetilde{\sigma}/(\partial\alpha\partial\lambda) < 0$.*

Proof. The proof is provided in Appendix B. $\square$

Under the assumptions of Proposition 2, $\tilde{\kappa}$ does not depend on energy or heterogeneity. Energy and heterogeneity, thus, shape indeterminacy only through $\widetilde{\sigma}$. We will use the wording that a feature of the model "raises the risk of indeterminacy" whenever $\widetilde{\sigma}$ falls with the corresponding parameter. Proposition 2 shows that the risk of indeterminacy rises in the share of energy in the economy ($\partial\widetilde{\sigma}/\partial\alpha < 0$), a result already familiar from Corollary 1. What the proposition adds to this is a view on how the fixed factor *interacts* with household heterogeneity. The proposition states that the larger the share of hand-to-mouth households (λ), the more does the share of energy in the economy raise the risk of indeterminacy ($\partial^2\widetilde{\sigma}/(\partial\alpha\partial\lambda) < 0$). The reason is simple. If the energy price rises, Foreign demand rises. This puts upward pressure on the inflation rate. The central bank raises the nominal (and real) interest rate. The larger the share of hand-to-mouth households is, however, the less interest sensitive is domestic demand. Thus, the larger λ the smaller the cut to domestic absorption and the more easily can the feedback loop be sustained. In sum, in an environment with limits to energy supply, a central bank that seeks to uniquely anchor expectations may need to react more strongly to inflation than otherwise and than prescribed by the Taylor principle.[10] This section's derivations assumed that energy is used in production only and not consumed directly. We turn to a quantitative assessment of the full model next. This allows to analyze further angles of the mechanism.

4 Numerical application – and policy implications

This section explores the model quantitatively by looking at a particular scenario: the scarcity of imported energy in Germany that arguably arose with the Russian invasion of Ukraine in February 2022. While, clearly, energy prices rose globally, too, what set Germany and several other European economies apart is that—on top of this global component—its energy infrastructure heavily depended on imported energy and Russian supply through pipelines, in particular. We use the numerical analysis to explore the energy-price-economic feedback loop and the scope for fiscal and monetary policy.

[10]Appendix B further shows that also in a closed economy, where all energy is owned domestically, $\iota = 1$, energy scarcity can amplify the risk of self-fulfilling fluctuations. This is so if the fixed factor renders hand-to-mouth households' income more procyclical.

4.1 Parameterization

With the parameterization we seek to illustrate the mechanism. So as to make the results easy to understand, we therefore limit the dimensions of heterogeneity. In particular, we target a steady state in which savers and hand-to-mouth households have the same level of income; an assumption that also underpins Proposition 2.

Table 1 provides the calibrated parameters. One period in the model is taken to be a quarter. We set $\beta = 0.995$, implying a two-percent annualized real rate of interest in

Table 1 Parameters of the baseline calibration

Parameter	Value	Description
Households		
β	0.995	Discount factor
σ	3	Inverse of intertemporal elasticity of substitution
χ	0.713	Disutility of labor supply
φ	3	Inverse of Frisch elasticity of labor supply
λ	0.22	Share of hand-to-mouth households
$\bar{e}$	0.125	Subsistence level of energy consumption
γ	0.255	Share of energy expenditures in consumption expenditures
η	0.1	Elasticity of substitution energy/goods in consumption
Firms		
ε	11	Elasticity of substitution different varieties of differentiated goods
ψ	389	Price adjustment costs
α	0.091	energy-intensity of production
θ	0.1	Elasticity of substitution between energy and labor in production
Energy supply and Foreign demand		
ι	0	Share of domestically owned energy
ξ_E	1.5	Steady state energy supply
$\mu_{F,1}$	0.25	Foreign's MPC out of energy revenues
$\mu_{F,2}$	0.02	Foreign's MPC out of savings
Government		
τ^y	0.1	Production subsidy
ν	0	Profit share, spenders
ϑ	0	Energy share, spenders (of domestic energy, see ι)
ϕ_Π	1.5	Response to inflation
τ_E^c	0.33	Energy price subsidy, households
τ_E^f	0.33	Energy price subsidy, firms

Notes: Parameters of the baseline calibration. See the text for details.

the steady state. We target an intertemporal elasticity of substitution of $1/\sigma = 1/3$, in line with estimates by Havránek (2015) and Best et al. (2020) or the heterogeneous-agent literature (Bayer, Born and Luetticke, 2022). The disutility of work $\chi = 0.713$ ensures that steady-state labor supply of both households is unity, a normalization. We calibrate $\varphi = 3$ so as to have a Frisch elasticity of labor supply of 1/3, within the standard range of values in the literature (Chetty et al., 2011). The share of hand-to-mouth households is $\lambda = 0.22$, following estimates for the euro area in Slacalek, Tristani and Violante (2020).

Calibrating the energy-related parts of the economy, we seek to have realistic shares of energy in both consumption and production. We do so to reflect the importance of imported energy in the economy. We set the subsistence level of energy consumption at 25 percent of households' energy consumption, $\bar{e} = 0.125$, following Fried, Novan and Peterman (2022). We set $\gamma = 0.255$ so that, in the steady state, households spend five percent of GDP on energy. The elasticity of substitution in consumption (between energy and the other goods) is $\eta = 0.1$, within the range reported in Bachmann et al. (2022).

Turning to production next, the own-price elasticity of demand is $\varepsilon = 11$, a conventional value. Price adjustment costs of $\psi = 389$ match a slope of the paper-and-pencil Phillips curve of 0.1. In a Calvo setting this would map into prices for *non-energy goods* being adjusted on average once a year. We aim for a cost of energy in production of 10 percent of GDP, giving $\alpha = 0.091$.[11] The elasticity of substitution in production (between energy and labor) is the same as in consumption, namely, $\theta = 0.1$; line with the range of estimates reported in Bachmann et al. (2022).

As regards supply of energy, we assume that all energy is imported, $\iota = 0$; this is reasonable for our German energy scenario. Note that the energy shares targeted also leave out domestic production (such as a small share of wind energy, for example). We normalize the supply of imported energy, ξ_E, so that firms' energy usage, E, takes on a value of unity in the steady state. As for the demand by Foreign of Home's goods, we set $\mu_{F,1} = 0.25$.

[11] Other papers have focused only on the imports of *natural gas* from Russia. Doing so, at 2022 prices, the shares above are around three and six percent of income for households and firms, respectively; confer, for example, Bachmann et al. (2022) and Pieroni (2023). The parameterization here, instead, considers a wider concept of energy scarcity and use, also accounting for other fossil energy imports (natural gas and coal, no oil). We calibrate the expenditure shares relative to GDP using German data on primary energy usage, see BDEW (2023). The relative share of households and firms is obtained from Eurostat's data on energy consumption by sector (product code: ten00124).

This means that for each additional euro in energy revenue, Foreign orders an additional quarter of a euro's worth of goods in Home. This renders Foreign's MPC of the same magnitude as the aggregate MPC out of income in the Home economy.[12] Next, we set the debt elasticity of demand to a value that is small but stabilizes net foreign assets at a long-run value of zero, $\mu_{F,2} = 0.02$.

We assume that the government sets $\tau^y = 0.1$ so that—in the steady state—the production subsidy undoes the distortion associated with firms' market power. We do so primarily for comparability with the propositions in Section 3. Further, we assume that savers receive all the profits in the economy ($\nu = 0$) and bear the burden of taxation whenever the government's funding needs deviate from the needs in steady state. Since savers are Ricardian, this assumption on tax incidence obviates a discussion of the timing of the tax incidence. Next, absent domestic ownership of energy (recall $\iota = 0$), we can fix an arbitrary value $\vartheta = 0$ (the spenders' share of non-existent domestically-owned energy endowments does not matter). As regards the energy-price subsidies, we assume that the same subsidy on the energy price applies to both households and firms. We set $\tau_E^c = \tau_E^f = 0.33$. These values are in the range that the literature explores; for example, Auclert et al. (2023). Note that the government subsidizes only that part of the price of energy that exceeds the normal (steady-state) level. Relative to the entire price, the subsidy, thus, is much smaller. Last, as regards monetary policy, and unless specified otherwise, we look at a response to core inflation of $\phi_\Pi = 1.5$, the conventional value.

4.2 Steady state

The parameterization from above was chosen such that both types of household have the same level of consumption, hours worked, and after-tax income in the steady state. Table 2 reports on the steady-state values. In the steady state, households' energy expenditures are five percent of GDP, as targeted. Similarly, firms' energy expenditures are ten percent

[12] Drechsel and Tenreyro (2018) discuss the effects of commodity price booms on commodity exporters. The increase in imports after an increase in commodity prices is sizable, speaking in favor of high MPCs out of energy revenues. Johnson, Rachel and Wolfram (2023) consider a unit MPC for an energy exporter, alluding to borrowing constraints and risks associated with accumulating financial assets (sanctions). Such considerations also apply to our scenario. We consider our parametrization here as conservative.

Table 2 Steady state under baseline parametrization

Variable	Value	Description	Variable	Value	Description
Households			Prices		
C	1.192	Consumption	$\Pi_G = \Pi$	1	Inflation
C_E	0.5	Energy cons.	P_E/P	0.121	Real energy price
C_G	0.864	Goods cons.	P_G/P	1.328	Real goods price
N	1	Labor supply	W/P	1.207	Real wage
Production			R	1.005	Gross nom. rate
Y_G	1	Output			
E	1	Energy in prod.			
D	0	Profits			
Λ	1.328	Real marginal costs			
		Implied ratios			
$\frac{P_E C_E}{P_E C_E + P_G C_G}$	0.05	Expenditure share of raw energy consumption over GDP			
$\frac{P_E E}{P_E C_E + P_G C_G}$	0.1	Energy input in production bill over GDP			
$\frac{P_E \xi_E}{P_E C_E + P_G C_G}$	0.15	Economy-wide expenditure on energy over value of GDP			

Notes: The steady state that corresponds to the baseline parameterization. Aggregates are denoted in per-capita terms. Since households are symmetric in the steady state, aggregates pertaining to the overall household sector also pertain to the individual household; for example, $C_H = C_S = C$.

of GDP, again as targeted. These two targets taken together mean that the cost of energy imports runs to the equivalent of 15 percent of the value of GDP.

4.3 The energy-price-activity loop

In this section, we numerically study the possibility that sunspot-driven fluctuations can arise in equilibrium even though monetary policy obeys the Taylor principle. First, we illustrate the mechanism. We then discuss the structural determinants. This complements the pencil-and-paper analysis presented earlier in Section 3. In all that follows, we focus on a first-order approximation of the model dynamics.

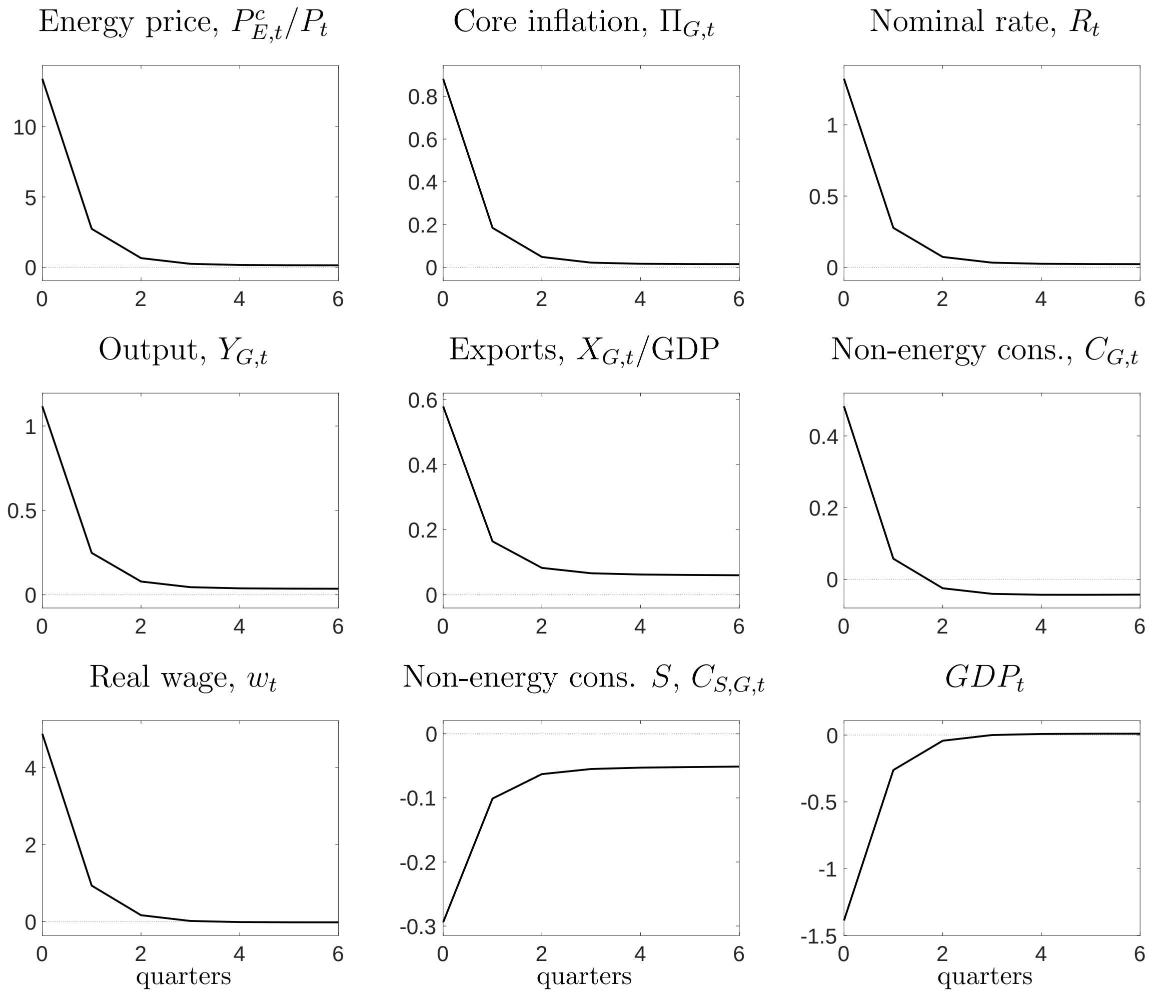
4.3.1 Indeterminacy amid the Taylor principle

Under the baseline parameterization, there is indeterminacy—even though (with $\phi_\Pi = 1.5$) monetary policy obeys the Taylor principle. For this finding, it matters that energy is scarce (that is, in fixed supply). In particular, with otherwise the same parameterization of the economy, if energy could be imported elastically at a given price, the conditions for determinacy would be entirely conventional: namely, the equilibrium would be determinate if and only if $\phi_{\Pi} > 1$. With energy being in fixed supply, however, even the response of $\phi_{\Pi} = 1.5$ embedded in the baseline parameterization does not suffice.

To see the mechanism at work, Figure 1 plots impulse responses to a sunspot shock in the baseline economy (with energy supply fixed and a monetary response to inflation of $\phi_{\Pi} = 1.5$). Exactly one explosive root is missing to satisfy the Blanchard-Kahn conditions. Thus, there is exactly one degree of indeterminacy and room for exactly one possible sunspot shock. To compute the impulse responses to this shock, we use the methodology of Bianchi and Nicolò (2021).[13] Theory pins down the persistence of the sunspot shock, but does not pin down its size (recall that we look at a linear approximation of the model). In the figure, we anchor the shock's size such that under the sunspot shock import prices for energy rise by 20 percent. With a 33 percent energy subsidy in place, energy prices for domestic customers (households and firms), rise by 13 percent (top row, left panel). The energy-price increase lasts for about a year (four quarters). On the back of higher energy costs, core inflation rises by about 0.8 percentage point (top row, center panel) reflecting that firms face higher production costs. In line with the Taylor rule, the central bank raises the nominal interest rate more than one for one with inflation (top row, right panel). A higher real interest rate means that savers' consumption falls (bottom row, center panel). Nevertheless, under the sunspot beliefs, the volume of production in Home rises by about one percent (second row, left panel). This is so for two reasons. First, under the sunspot belief of higher energy prices, Foreign's revenue rises. In our calibration, Foreign immediately uses 25 percent of the rise in revenue for buying goods from Home (second row, center panel). A further effect, directly linked to the heterogeneity of households is that the domestic demand for the consumption goods as a whole does not fall until some time after the shock (second row, right panel). This reflects that while savers retrench their consumption demand (interest rates rise and their income falls), the hand-to-mouth households' budget is supported by the labor-market effects in this scenario. Namely, labor demand rises (not shown) and so does the real wage (bottom row, left panel).

[13] While the exposition of the model did not spell out fundamental shocks, such as monetary shocks, the reader may think of the sunspot shock as not being correlated with potential fundamental shocks.

Figure 1 Sunspot shock amid targeting core inflation



Notes: Impulse responses to a sunspot shock that raises import prices for energy by 20 percent on impact. The central bank responds to core inflation, with $\phi_\Pi = 1.5$. Scaling: all response are scaled to give percentage deviations from steady state. The exception is the response of exports (percent of steady state GDP). Also, interest rates and inflation rates are in annualized percentage points.

Labor demand rises because, on the one hand, a higher energy price means that firms substitute toward labor. On the other hand, also the higher level of production comes with higher labor demand. The real wage rises by about four percent. It is important to note, however, that in all these dynamics GDP—as opposed to the level of production—falls (bottom row, right panel). The reason why GDP falls is simple: even though production rises, value added falls on the back of higher costs for energy imports.

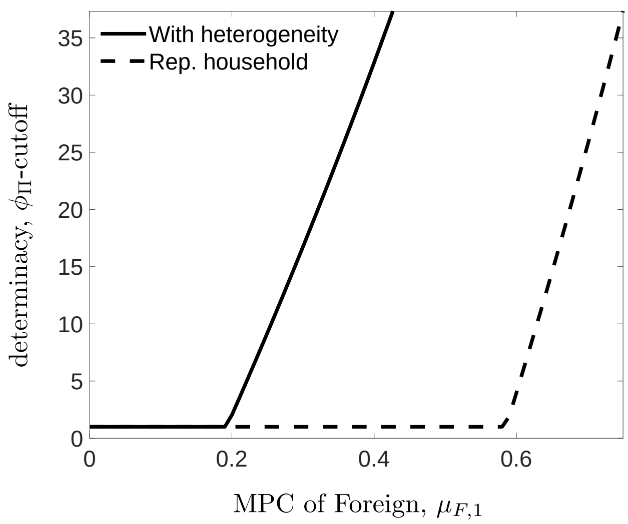
Figure 1 showed the response to a sunspot shock when the central bank's response coefficient was $\phi_\Pi = 1.5$. The scope for indeterminacy is wider than this, however. Namely, indeterminacy arises whenever $\phi_\Pi < 9.23$. This cutoff for the response of the interest rate to inflation is notably larger than the cutoff that the Taylor principle prescribes. Next, we decompose what drives indeterminacy, starting from the baseline parameterization.

4.3.2 What matters for the feedback loop?

This section seeks to further clarify the mechanism by highlighting the non-monetary dimensions of the economy that are important in shaping the feedback loop.

Marginal propensities to consume (MPCs). Heterogeneity plays a crucial role for the existence and the size of the feedback loop. Namely, when agents coordinate on non-fundamental beliefs of high energy prices, in the scenario shown above not only aggregate economic activity changes but also the distribution of incomes. In particular, incomes are distributed from agents with a lower MPC toward agents with a higher MPC. In the scenario underlying Figure 1 output rises, so that world-wide incomes rise. This rise in world-wide incomes is unevenly distributed, however. It accrues to Foreign due to windfall gains from higher energy prices. And it accrues to the spender households in Home due to higher wages and fiscal redistribution. This comes at the expense of the incomes of saver households who see the profits of firms fall amid rising input costs and falling markups. The importance of the relative MPCs is illustrated in Figure 2. The

Figure 2 Marginal propensities to consume and the determinacy cutoff



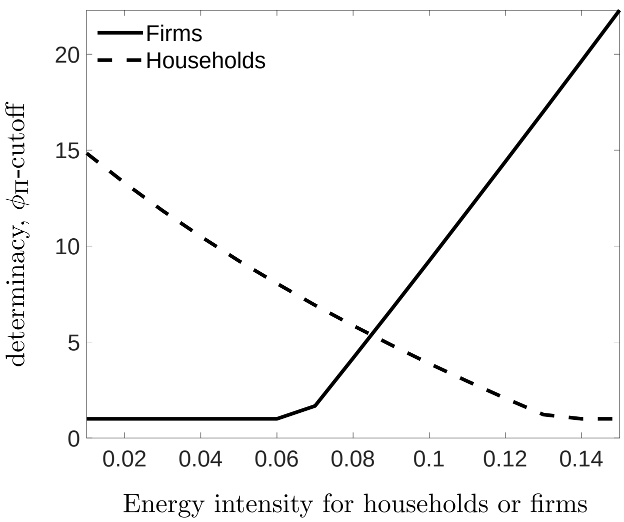
Notes: The figure plots the determinacy cutoff for the baseline policy rule (y-axis), varying Foreign's MPC, $\mu_{F,1}$ (x-axis). The other parameters are fixed at the baseline parameterization, cf. Table 1. The equilibrium is unique whenever ϕ_Π is larger than the cutoff values shown here. Solid line: baseline parameterization (apart from $\mu_{F,1}$). Dashed line: representative household in Home, $\nu = \vartheta = \lambda$.

y-axis of the figure shows the determinacy cutoff. For any value of ϕ_Π (the monetary response to inflation) larger than the cutoff (above the respective lines), the equilibrium will be unique. For any value lower than that, the feedback loop arises. The x-axis varies Foreign's MPC out of energy revenue, $\mu_{F,1}$. Focus first on the solid line, which shows how the determinacy cutoff varies in the baseline model, that has the said two types of households. The Taylor principle holds if Foreign's MPC does not exceed 0.19. Beyond that, the determinacy cutoff rises quickly (the baseline parameterization has a value of $\mu_{F,1} = 0.25$). Next, focus on the dashed line. This illustrates the role of heterogeneity within the Home economy. Namely, the economy underlying the dashed line here is calibrated the same way as the baseline, but does not feature any income heterogeneity in the Home economy ($\nu = \vartheta = \lambda$). The difference between the solid line and the dashed line, thus, provides a measure of how important the heterogeneity *within* the Home country is. Clearly, heterogeneity matters. If the Home economy had representative households, all of which had access to financial market, the feedback loop would arise only when the Foreign MPC exceeds 0.58. The kink of that dashed line (representative household in Home), thus, occurs at a value for the MPC of Foreign that is about three times as large as in the baseline economy (two types of households in Home).

Similar considerations would apply as regards the ownership of energy. In the baseline, energy is owned entirely by Foreign. Since Foreign has the same MPC as the average household in the Home economy, the results would be very similar (and the feedback loop would continue to arise) if energy was owned in Home rather than Foreign ($\iota = 1$), provided that energy ownership would not be too unevenly distributed in Home (results are not shown in the Figure above). In this alternative scenario, and with otherwise the same parameters as in the baseline parameterization, one can show that the Taylor principle remains violated whenever the hand-to-mouth households' share in domestic energy ownership is at least $\vartheta > 0.19$. *Vice versa*, if the gains from higher energy prices were to accrue disproportionately to savers (ϑ small), the Taylor principle is alive and well; note that these chime well with the message that Figure 2 gives.

Supply shortages here affect production, not only consumption. Supply shortages may primarily affect the supply of goods for consumption or of factors of production. In the baseline economy, they affect both since energy is used in consumption and production. We next illustrate how this shapes the feedback loop. Figure 3 takes our baseline

Figure 3 Energy intensity and the determinacy cutoff



Notes: The figure plots the determinacy cutoff for the baseline policy rule (y-axis), varying the steady-state share of energy expenditures of households or firms (x-axis). Solid line: the expenditures of firms vary, while expenditures of households remain at the baseline value of 5 percent of GDP. Dashed line: the expenditures of households vary, keeping the expenditures of firms at the steady-state share of 10 percent of GDP. Only parameters α, γ, and $\bar{e}$ (the latter to keep the subsistence share constant) vary. All other parameters remain fixed at their values in the baseline parameterization, cf. Table 1.

scenario, which respects that energy is used both for production and consumption. Keeping all other parameters at the same values as in the baseline parameterization, the figure adjusts three parameters that govern the steady-state share of energy in expenditures for production and consumption, respectively, namely α, and γ, and $\bar{e}$. We adjust these parameters so that in the steady state the share of energy in production varies, keeping the share of energy in consumption constant, or the other way around.[14] The x-axis of the figure shows the size of the respective energy share that varies (expenditures as a share of GDP). The y-axis shows the determinacy cutoffs. The solid line varies the share of energy use in production. The Taylor principle is violated as soon as firms' expenditure share of energy exceeds six percent of GDP. This shows that for the feedback loop to emerge, it is important that the supply constraints affect production, and sufficiently much so.

[14] In all these scenarios, we make sure that while households' total energy consumption varies, the share of households' energy consumption that is due to subsistence energy-consumption remains the same.

Then, due to energy's use in production, higher energy prices set in train a redistribution of incomes toward agents with a higher MPC than the owners of firms have. Therefore, aggregate demand can be high, validating the higher energy prices.

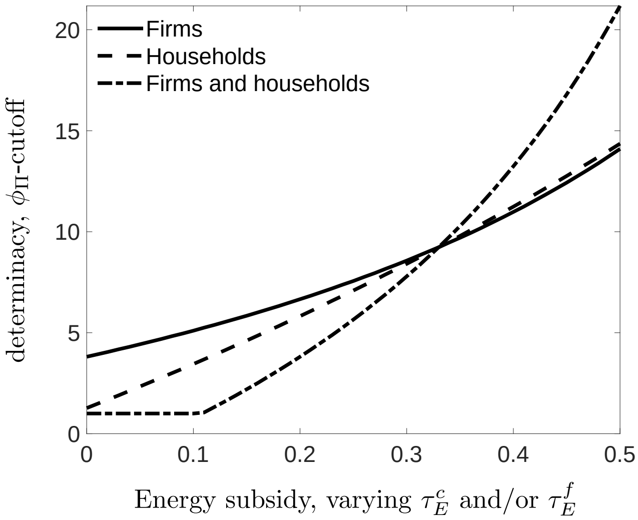
The dashed line varies the consumption expenditures for energy. The figure shows that if households use less energy than in the baseline, the determinacy cutoff rises. In other words, energy use in consumption dampens the feedback loop. A larger share of energy in households' expenditures in turn lowers the determinacy cutoff toward the one prescribed by the Taylor principle. Whenever the expenditure share of energy for consumption exceeds 13 percent, the Taylor principle is restored entirely. The use of energy in consumption affects the feedback loop for two reasons. First, if households substitute away from energy, the supply of energy available for production rises. Essentially, the supply of energy to firms becomes more elastic. This means that labor demand and wages rise to a lesser extent than if energy is used in production only. Second, domestically, rising energy prices act like a tax on households' effective incomes, savers' and hand-to-mouth households' alike.

4.3.3 Fiscal policy and the feedback loop

The feedback loop arises when high demand for goods comes with high energy prices, and if these in turn do not substantially dampen demand. In the baseline scenario, fiscal policy intervenes in the energy market. It subsidizes the use of energy at the margin for both households and firms; namely, through the subsidies τ_E^c and τ_E^f.

Figure 4 illustrates the importance of the subsidies. The figure keeps the parameters of the baseline, but varies the subsidies for households' and/or for firms' use of energy. Focus on the dashed line, first. The scenario shown by that line varies the subsidy to households (τ_E^c, x-axis) but keeps the subsidy to firms (τ_E^f) and all other parameters at their baseline values. As in the earlier figures, the y-axis plots the threshold for determinacy. Absent the subsidy to households, the Taylor principle barely is violated (left end of the dashed line). Thereafter, the determinacy cutoff rises steeply. The energy subsidy to households, in sum, provides considerable support to the feedback loop. This makes sense because the

Figure 4 Energy subsidies and the determinacy cutoff



Notes: The figure plots the determinacy cutoff for the baseline policy rule (y-axis), varying energy price subsidies – either τ_E^c or τ_E^f or both at the same time (x-axis). The other parameters are fixed at the baseline parameterization, cf. Table 1. The baseline value for both τ_E^c and τ_E^f was 0.33.

subsidy makes households less inclined to substitute away from energy. This keeps the energy price elastic to output and strengthens the redistribution of incomes, to both hand-to-mouth households (since the subsidies are financed by savers) and Foreign. Compared with this, the energy subsidy to firms alone appears of lesser importance: The solid line in Figure 4 varies τ_E^f, leaving all other parameters at their baseline values. Even absent such subsidies on the production side (left end of the solid line), the central bank would still need to respond strongly to inflation to ensure determinacy ($\phi_\Pi > 3.81$). In fact, even in theory, the effect of subsidies is more ambiguous for firms (that is, in production) than for households (that is, in consumption). On the one hand, the subsidy on the production side also keeps energy demand high, leading to an increase in the import price and a redistribution to Foreign. On the other hand, an energy subsidy to firms dampens the rise in marginal costs. Firms shift to labor to a lesser extent, thereby dampening the rise in wages and the domestic redistribution that underpins the feedback loop. Of course, the effect on outcomes is stronger still (the gradient steeper) when subsidies to both firms and households vary simultaneously (dashed-dotted line). In the baseline scenario, fiscal policy aimed at countering the burden of higher energy prices through subsidies is a key element in supporting the feedback loop.

4.4 Monetary policy options

We have presented a scenario in which self-fulfilling fluctuations can arise. The environment was one in which a production factor is in fixed supply. At the core was that these fluctuations redistribute income from agents with a low MPC to agents with a higher MPC. Several conditions had to be met: the supply-constrained factor had to be a sufficiently important part of firms' costs, labor markets sufficiently tight (so that wages respond),[15] and fiscal policy had to be supportive of such dynamics. All of these dimensions could, in principle, be addressed with targeted policies (energy, fiscal, or labor-market policy). Here we take these policies as given or absent and, instead, focus on what monetary policy alone could achieve. The conventional wisdom is that central banks should "look through" energy-price shocks. Instead, all the policies that we discuss below, which successfully control the feedback loop, have in common that, one way or another, they make the central bank lean against the energy-price increase that is part of the feedback loop. The reason why the central bank achieves determinacy by apparently leaning against energy prices is that higher energy prices in our scenario are a symptom rather than an exogenous "shock." Namely, the sharply higher energy prices reflect that demand meets a scarce input.

Targeting headline inflation. The results that we have presented so far looked at a response to core inflation, in line with the idea of "looking through energy-price movements." Core inflation accounts for energy prices only indirectly, that is, only to the extent that higher input prices pass through to goods prices. Targeting headline consumer price inflation, instead, puts some direct weight on energy prices; namely, the weight that energy receives in the consumption basket. If that weight is large enough, a response to headline inflation can be sufficient to ensure determinacy even with a conventional monetary response. In the baseline parameterization, this additional weight suffices: If the central bank responds to headline inflation instead of core inflation, determinacy prevails whenever $\phi_\Pi > 1$. What is important for the strategy to work is that *consumer price*

[15] This is a loose interpretation of labor supply being sufficiently inelastic and wages flexible.

inflation reflects the cost pressures of *firms* to a sufficient extent. In this light, a response to headline inflation restores the Taylor principle only if households spend more than 3.5 percent of their income on energy to start with (the baseline has 5 percent), and if fiscal policy does not shield headline inflation too much from wholesale energy price pressures (the subsidy to consumers must be smaller than $\tau_E^c = 0.45$; the baseline has $\tau_E^c = 0.33$). In this sense, targeting headline consumer price inflation works in the baseline parameterization, but is not fail-safe.

Targeting input prices rather than consumer prices. The energy-price-activity loop is at work if goods prices are rigid. Rigidity means that firms do not fully pass rising costs on to domestic or foreign consumers. This suggests that the central bank might as well try to respond to the rising nominal marginal costs directly; namely, to target input-price inflation, (12). At the baseline parameterization this, too, restores the Taylor principle: any response with $\phi_\Pi > 1$ would imply determinacy. What is more, numerically, this is so independent of the use of energy in consumption or the extent of subsidies provided.[16] Leaning against the rise in input prices means both that the central bank leans against the energy-price fluctuations and that it leans against the wage increases that are part of what looks like a "second-round effect" of higher energy prices in Figure 1. Of course, the central bank might just as well target energy prices directly. Indeed, for example, if the central bank were to continue to react to core inflation with a weight of $\phi_\Pi = 1.5$ and added a seemingly small additional weight of above 0.01 on energy-price inflation, $P_{E,t}/P_{E,t-1}$, this would ensure determinacy as well. Once more, determinacy may require to explicitly or implicitly lean against energy prices.

Response to economic activity. In the feedback loop, higher energy prices reflect high demand in a supply-constrained environment. A further option for the central bank would, thus, seem to be to respond not only to inflation but also to a measure of economic activity. The energy-price-activity loop scenario that we outlined above sees higher energy prices go in hand with higher employment (and output) but lower GDP (since a larger claim to

[16] We could not find a scenario where targeting input prices fails to restore the Taylor principle.

production accrues to Foreign). All this happens at a time when energy prices are high and consumption low. Since different measures of economic activity give different signals, the choice of measure of activity matters. A central bank that responds in a conventional way to GDP (easing when GDP is low), for example, exacerbates the feedback loop. The reason is that leaning against low GDP *stimulates* economic activity and the energy price still further. Instead, a central bank that leans against the rise in employment (or production) dampens the feedback loop. Quantitatively, consider a central bank that responds to core inflation with the baseline weight of $\phi_\Pi = 1.5$. Next to this, let the central bank respond to output (add a coefficient ϕ_{Y_G} to the Taylor rule). At otherwise the baseline parameters, a coefficient on output of at least $\phi_{Y_G} > 0.61$ restores determinacy.[17]

Real rate rules. Holden (2022) recently proposed that, in a wide range of environments, if the central bank responds to a market measure of the real rate of interest and not only to inflation, the Taylor principle is restored. The same is true in the current setting. Let r_t be the ex-ante real rate of interest. If instead of applying (10) the central bank follows

$$R_t = r_t \left(\Pi_{G,t}\right)^{\phi_\Pi}, \text{ with } \phi_\Pi \geq 0 \text{ and with } r_t := R_t/[\mathbb{E}_t \Pi_{G,t+1}], \tag{17}$$

determinacy is restored for any $\phi_\Pi > 1$.[18] The economics are as follows: In the course of the energy-price-activity feedback loop, the real incomes received by savers fall. Savers will, thus, be inclined to borrow, which puts upward pressure on the real rate of interest. A central bank that follows rule (17), therefore, leans more firmly against the feedback loop than under the more conventional rule (10).

What all the above monetary-policy alternatives have in common is the following: if conditions are such that there is the possibility of an energy-price-activity feedback loop, to avoid self-fulfilling fluctuations the central bank may have to be willing to *raise* interest rates firmly when energy prices rise, even though there is a recession as measured by

[17] Compare this with a response in the rules of Taylor (1993) and Taylor (1999), which have output coefficients of 0.125 and 0.25, respectively.

[18] As explained in Holden (2022), (17) implies a difference equation in $\Pi_{G,t}$ only, that has a unique stable solution if and only if $\phi_\Pi > 1$. Removing the nominal indeterminacy removes the real indeterminacy.

consumption or GDP. Higher energy prices here are a symptom of high demand in a supply-constrained environment. The central bank needs to be committed to *raising* the real rate sufficiently much so as to lean against demand.

5 Conclusions

The current paper has worked with a New Keynesian business cycle model with a fixed factor of production and heterogeneous households. We have applied the model to the environment of energy scarcity that some European economies, Germany in particular, experienced after the Russian invasion of Ukraine in 2022. The fixed factor, thus, was assumed to be imported energy. Energy was supplied inelastically by a foreign economy. The price at which energy traded moved flexibly to align domestic demand with the fixed supply.

We showed that this environment can give rise to an energy-price-activity feedback loop that supports the following as a sunspot equilibrium: a high energy price, high core inflation, high interest rates *and* high economic activity as measured by employment, all at the same time; but a fall in gross domestic product. In this loop, high energy prices reflect high demand in a supply-constrained environment. High energy prices mean that external demand is high. The substitution of production away from energy and toward labor means that domestic demand falls little, even if the central bank raises interest rates in a conventional way with inflation.

The channel provides a rationale for why—in a scarce-energy situation—the central bank may precisely *not* follow the conventional wisdom to "see through energy shocks" or to disregard movements in energy prices on the basis that such prices are flexible. Instead, the central bank may rather want to *raise* interest rates when energy prices rise—even if this is recessionary. In the same vein, the central bank may want to focus on headline inflation rather than core inflation, may want to overweight the energy price in its inflation considerations, or may want to be more active in trying to slow economic activity even if the gross domestic product is already falling.

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Online Appendix to:

Limited (energy) supply, sunspots, and monetary policy

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— Not for publication —

A Paper-and-pencil model without heterogeneity

This appendix provides the proofs of Section 3 for the representative-household variant of the model.

A.1 Proof of Proposition 1

This appendix provides the proof for Proposition 1. The proof is straightforward and the steps well-known in the New Keynesian literature. The model is given by equations (14) and (15), repeated here for convenience (recalling that $\widehat{\Pi}_{G,t} = \widehat{\Pi}_t$).

$$\widehat{Y}_{G,t} = \mathbb{E}_t \widehat{Y}_{G,t+1} - \frac{1}{\widetilde{\sigma}} \left[\phi_\Pi \widehat{\Pi}_t - \mathbb{E}_t \widehat{\Pi}_{t+1} \right], \qquad \text{with } \widetilde{\sigma} := \frac{\sigma}{1-\alpha} \frac{1-\alpha\left[1+\varphi+1/\theta\right]}{1+\alpha(\sigma-1)}, \tag{18}$$

$$\widehat{\Pi}_t = \beta \mathbb{E}_t \widehat{\Pi}_{t+1} + \widetilde{\kappa} \widehat{Y}_{G,t}, \qquad \text{with } \widetilde{\kappa} := \frac{\varepsilon}{\psi} \frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)}. \tag{19}$$

The proposition states the importance of the signs of $\widetilde{\sigma}$ and $\widetilde{\kappa}$ which are determined by

$$\begin{aligned} \operatorname{sgn} \widetilde{\sigma} &= \operatorname{sgn} \frac{\sigma}{1-\alpha} \frac{1-\alpha\left[1+\varphi+1/\theta\right]}{1+\alpha(\sigma-1)} = \operatorname{sgn}\left(1-\alpha\left[1+\varphi+1/\theta\right]\right), \\ \operatorname{sgn} \widetilde{\kappa} &= \operatorname{sgn} \frac{\varepsilon}{\psi} \frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)} = \operatorname{sgn}\left(\sigma+\varphi-\alpha/\theta(\sigma-1)\right), \end{aligned}$$

or

$$\begin{aligned} \widetilde{\sigma} > 0 \qquad &\longleftrightarrow \qquad 1 - \frac{\alpha}{\theta} > \alpha(1+\varphi), \\ \widetilde{\kappa} > 0 \qquad &\longleftrightarrow \qquad 1 - \frac{\alpha}{\theta} > -\frac{1}{\sigma}\left(\varphi + \frac{\alpha}{\theta}\right), \end{aligned}$$

where $\alpha(1+\varphi) > 0$ and $-\frac{1}{\sigma}\left(\varphi + \frac{\alpha}{\theta}\right) < 0$. Hence, whenever $\widetilde{\sigma} > 0$, also $\widetilde{\kappa} > 0$. For $\widetilde{\sigma} < 0$, we can still have either $\widetilde{\kappa} > 0$ or $\widetilde{\kappa} < 0$.

Intuition for the sign of $\widetilde{\kappa}$

The sign of $\widetilde{\kappa}$ is determined by the sign of the term $\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)}$, which gives the elasticity of marginal costs with respect to output. The denominator is positive. We, thus, focus on the numerator ($\sigma+\varphi-\alpha/\theta(\sigma-1)$), the sign of which is ambiguous. The first two summands in the numerator are conventional. If energy prices were to move in lock-step with wages, these terms would give the entire effect of output on marginal costs. The terms are the wealth effect on labor supply and the compensation for the disutility of work that would come with rising output alone (rising employment). Both of these terms are unambiguously positive. What could make the sign of $\widetilde{\kappa}$ switch, instead, is the term $-\alpha/\theta(\sigma-1)$. This term captures the effect on marginal costs of the excess sensitivity of the energy price with respect to output (the rise of energy prices in excess of movements in the wage). The energy price is given by $\widehat{q}_t = \widehat{w}_t + \frac{1}{1-\alpha}\widehat{Y}_{G,t}$. Real marginal costs are given by $\widehat{\Lambda}_t = (1-\alpha)\widehat{w}_t + \alpha\widehat{q}_t$ and, hence, by $\widehat{\Lambda}_t = \widehat{w}_t + \frac{\alpha/\theta}{1-\alpha}\widehat{Y}_{G,t}$. The less substitutable the production factors are (the smaller θ), the larger is the excess sensitivity of energy prices with respect to output. In turn, the larger the share of energy is in production (the larger α), the more this matters for marginal costs. This excess sensitivity has two countervailing effects on marginal costs: a direct effect and an indirect effect that runs through the wage. The direct effect means higher output comes with higher marginal costs still. The indirect effect works in the opposite direction. Namely, for a given level of output, a rise in energy costs reduces household consumption. Through the wealth effect on labor supply, this reduces wages and marginal costs. The larger σ is, the larger is the wealth effect on labor supply and, thus, the larger is this opposite force on marginal costs. For a strong-enough wealth effect on labor supply (large-enough σ) and strong-enough excess sensitivity of energy prices (large enough α/θ), the sign of the slope of the Phillips curve could, thus, invert.

Intuition for the sign of $\widetilde{\sigma}$

The intuition for the sign of $\widetilde{\sigma}$ is more straightforward. The sign of $\widetilde{\sigma}$ is determined by the sign of $A := 1 - \alpha\,[1+\varphi+1/\theta]$. The term reflects the comovement of aggregate consumption with economic activity. Namely, by market clearing $\widehat{Y}_{G,t} = (1-\alpha)\widehat{c}_t + \alpha\widehat{q}_t$., so that $\widehat{c}_t = \frac{1}{1-\alpha}\left[\widehat{Y}_{G,t} - \alpha\widehat{q}_t\right]$. Since $\widehat{q}_t = \frac{\sigma+\varphi+1/\theta}{1+\alpha(\sigma-1)}\widehat{Y}_{G,t}$, combining terms, we have that $\widehat{c}_t = \frac{1-\alpha[1+\varphi+1/\theta]}{(1-\alpha)(1+\alpha(\sigma-1))}\widehat{Y}_{G,t}$. α is the share of energy in production and, therefore, the share of output exported in steady state. If energy prices were constant altogether and all factor inputs linear in output, $A = 1-\alpha$. The term in the squared bracket in term A reflects that input prices move disproportionately with output. The reason is that households' disutility of labor increases in output. If energy prices were to move one-to-one with wages $A = 1-\alpha(1+\varphi)$. Term $1/\theta$ in the square bracket in turn captures once again the excess sensitivity of energy prices to movements in output.

Thus, the IS curve inverts if $\alpha\,[1+\varphi+1/\theta] > 1$, or, in words, if energy is an important input factor (high α), labor supply of households is sufficiently inelastic (high φ), and if energy is hard to substitute (high $1/\theta$). Suppose that firms seek to produce an additional unit of output $Y_{G,t}$. Since energy is in fixed supply, labor supply needs to expand. At the same time, both energy and labor are needed in production due to the low elasticity of substitution. The lower the elasticity of substitution (low θ) the more does the energy price increase. Similarly, when households' labor supply is inelastic ($\varphi > 0$), the wage increases and firms would prefer to use energy instead of labor: again, the energy price rises. Up to first order, firms' profits (here labelled Γ_t) are given by

$$\Gamma_t = (1-\alpha)\widehat{N}_t - (1-\alpha)(\widehat{w}_t + \widehat{N}_t) - \alpha\widehat{q}_t = -(1-\alpha)\widehat{w}_t - \alpha\widehat{q}_t.$$

Incomes accruing to domestic households are given by profits and wages, up to first order these incomes evolve as

$$\Gamma_t + \widehat{w}_t + \widehat{N}_t = \alpha[\widehat{w}_t - \widehat{q}_t] + \frac{1}{1-\alpha}\widehat{Y}_{G,t}.$$

This means that if energy prices rise sufficiently steeply when output rises, the incomes of domestic households can fall even though wages may still rise. In turn, an increase in the real rate of interest could *crowd in* domestic consumption by depressing energy costs (and, thus, the share of output transferred abroad).

Determinacy cutoffs

Write the model in Blanchard and Kahn (1980) form:

$$\begin{bmatrix} 1 & 1/\widetilde{\sigma} \\ 0 & \beta \end{bmatrix} \times \mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \phi_\Pi/\widetilde{\sigma} \\ -\widetilde{\kappa} & 1 \end{bmatrix} \times \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix}$$

or, alternatively

$$\mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}} & \frac{1}{\widetilde{\sigma}}\phi_\Pi - \frac{1}{\beta}\frac{1}{\widetilde{\sigma}} \\ -\frac{1}{\beta}\widetilde{\kappa} & \frac{1}{\beta} \end{bmatrix}}_{:=A} \times \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix}$$

There are two nonpredetermined variables. So there will always be bounded equilibria. There is a locally unique bounded equilibrium iff either (cf. Woodford, 2003, p. 670):

- Case a): $det(A) > 1$, $det(A) - tr(A) > -1$ and $det(A) + tr(A) > -1$, or
- Case b): $det(A) < 1$, $det(A) - tr(A) < -1$ and $det(A) + tr(A) < -1$.

Here, $det(A) = \left[\frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\phi_\Pi\right]$ and $tr(A) = \left[1 + \frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\right]$.

Proof of the proposition's item 1). Suppose that $\widetilde{\sigma} > 0$ and $\widetilde{\kappa} > 0$. Then the determinacy conditions are as in the standard closed economy New Keynesian model. More in detail, $det(A) > 1$ and $tr(A) > 0$, so that Case a) applies. The condition that may bind is $det(A) - tr(A) > -1$, which leads to the conventional determinacy condition $\phi_\Pi > 1$.

Proof of the proposition's item 1) c'td. Suppose that $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} < 0$. Then the determinacy conditions are as in the standard closed-economy New Keynesian model. Again, in this case $det(A) > 1$ for any $\phi_\Pi > 0$. Thus, we need to check Case a) again. $tr(A) > 0$, so that $det(A) + tr(A) > -1$ always. So, what we need for determinacy is $det(A) - tr(A) > -1$. Or, equivalently $\left[\frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\phi_\Pi\right] - \left[1 + \frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\right] = \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}[\phi_\Pi - 1] - 1 > -1$, or, once more, $\phi_\Pi > 1$.

Proof of the proposition's item 2). By assumption for this case, $\widetilde{\sigma} < 0$, $\widetilde{\kappa} > 0$. This is when non-standard determinacy regions arise. In this case, *two* regions can arise.
Focus on the set of conditions for case a) first. $det(A) = \left[\frac{1}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}\phi_\Pi\right] > 1$ can be achieved by setting $\phi_\Pi < \frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1)$, where $\frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1) > 0$ since $\frac{\widetilde{\sigma}}{\widetilde{\kappa}} < 0$. The second condition can be achieved by setting $\phi_\Pi < 1$. Finally, the third condition can be achieved by setting $\phi_\Pi < -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1$. Hence, this determinacy region exists if there is a $\phi_\Pi \geq 0$ such that

$$\phi_\Pi < \min\left(\frac{\widetilde{\sigma}}{\widetilde{\kappa}}(\beta - 1),\ 1,\ -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right)$$

Note that, for $\beta \to 1$, this determinacy region disappears.
Focus on the set of conditions for case b) next. $det(A) < 1$ can be achieved for $\phi_\Pi \geq 0$ since $\frac{\widetilde{\sigma}}{\widetilde{\kappa}} < 0$. For $det(A) - tr(A) < -1$, we need $\frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}[\phi_\Pi - 1] - 1 < -1$, meaning $\phi_\Pi > 1$. For $det(A) + tr(A) < -1$, we need $1 + \frac{2}{\beta} + \frac{1}{\beta}\frac{\widetilde{\kappa}}{\widetilde{\sigma}}(\phi_\Pi + 1) < -1$, meaning $\phi_\Pi > -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1$. So that both $det(A) \pm tr(A) < -1$, therefore we need

$$\phi_\Pi > \max\left(1, -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1\right),$$

or for $\beta \to 1$, $\phi_\Pi > \max\left(1, -4\widetilde{\sigma}/\widetilde{\kappa} - 1\right)$. This is the cutoff mentioned in Proposition 1, equation (16). □

A.2 Proof of Corollary 1

Consider case 2) of Proposition 1, that is, $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$. For those restrictions to be satisfied, we require that

$$\begin{array}{lll} \text{if } 1 - \alpha(1+\varphi) \leq 0 & \text{then} & \alpha\frac{\sigma-1}{\varphi+\sigma} < \theta \\ \text{if } 1 - \alpha(1+\varphi) > 0 & \text{then} & \alpha\frac{\sigma-1}{\varphi+\sigma} < \theta < \frac{\alpha}{1-\alpha(1+\varphi)}. \end{array} \tag{20}$$

The lower bound on θ comes from $\widetilde{\kappa} > 0$. The lower bound puts limits on the strength of the wealth effect on labor supply, discussed on top of page App – 2. For $\sigma \leq 1$, the lower bound never binds ($\theta > 0$ in any case). If the share of energy is large enough and labor supply sufficiently inelastic (the top case), labor and energy could be infinitely substitutable; and, nevertheless, the IS curve inverts, compare the text on the bottom of page App – 2. Otherwise, if the share of energy in production, α, is smaller or the labor supply elasticity takes on intermediate values, the elasticity of substitution between energy and labor must not be too large to see the IS curve invert. The reason is that, then, energy prices must be sufficiently elastic to generate inversion. Corollary 1 looks at the case $\theta = \alpha$. For this case, the inequalities above are satisfied for any $\alpha > 0$, $\sigma > 0$, and $\varphi > 0$.
The determinacy threshold for case two is tighter than the Taylor principle would require

if $\overline{\phi}_\Pi > 1$, where $\overline{\phi}_\Pi$ is defined as the threshold

$$\overline{\phi}_\Pi := -2(1+\beta)\frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1 = 2(1+\beta)\frac{\sigma}{\varepsilon/\psi}\frac{\alpha}{1-\alpha}\frac{1+\varphi+1/\theta-1/\alpha}{\varphi+\sigma-\alpha/\theta(\sigma-1)} - 1.$$

For a tighter determinacy criterion than the Taylor principle ($\overline{\phi}_\Pi > 1$), thus, we have

$$(1+\beta)\frac{\sigma}{\varepsilon/\psi}\frac{\alpha}{1-\alpha}\frac{1+\varphi+1/\theta-1/\alpha}{\varphi+\sigma-\alpha/\theta(\sigma-1)} > 1. \tag{21}$$

Given the parameter restrictions on θ in (20), we have that

$$1+\varphi+1/\theta-1/\alpha > 0 \qquad \text{and} \qquad \varphi+\sigma-\alpha/\theta(\sigma-1) > 0,$$

such that we can rearrange condition (21) to

$$1 > \frac{1}{1+\beta}\frac{\varepsilon/\psi}{\sigma}\frac{1-\alpha}{\alpha}\frac{\varphi+\sigma-\alpha/\theta(\sigma-1)}{1+\varphi+1/\theta-1/\alpha}, \tag{22}$$

where each single fraction is strictly positive. With this, we have the following corollary, which nests corollary A.1 when $\alpha = \theta$.

Corollary A.1. *Consider the same conditions as in Proposition 1 in the main text. In addition, the parameter restrictions in (20) hold. This implies case 2) of the proposition (potentially unconventional with $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$). We have the following: An arbitrary response $\phi_\Pi > 1$ ensures determinacy if and only if the following inequality holds:*

$$\Omega := \frac{1}{2}\frac{\varepsilon/\psi}{\sigma}\frac{1-\alpha}{\alpha}\frac{\varphi+\sigma-\alpha/\theta(\sigma-1)}{1+\varphi+1/\theta-1/\alpha} \geq 1.$$

We can study the effect of the parameters on the emergence of self-fulfilling fluctuations. The lower Ω, the more likely it is that the Taylor principle is not sufficient. Recall that, given the parameter restrictions (20) due to studying case 2) of Proposition 1, both the numerator as well as the denominator of Ω are positive. The following holds. (i) $\partial\Omega/\partial(\varepsilon/\psi) > 0$, i.e., the flatter the Phillips curve, absent energy-price feedback, the more likely the Taylor principle is violated. (ii) $\partial\Omega/\partial\sigma < 0$, i.e., the more unwilling households are to substitute intertemporally, the more likely the Taylor principle is violated. (iii) $\partial\Omega/\partial\alpha < 0$ (under very mild regularity conditions), i.e., the larger the share of energy, the more likely the Taylor principle is violated. (iv) $\partial\Omega/\partial(1/\theta) < 0$, i.e., the more inelastic energy demand by firms, the more likely the Taylor principle is violated.

As to the qualitative statements in Corollary 1 in the main text, they emerge when applying the restriction $\alpha = \theta$ to (22). In that case, $\Omega = \frac{1}{2}\frac{\varepsilon/\psi}{\sigma}\frac{1-\alpha}{\alpha}$, making the sign of the respective derivatives obvious.

B Paper-and-pencil model with heterogeneity

This Appendix provides proofs for the heterogeneous-household version of the model. Appendix B.1 provides a proposition for the case when all energy is owned domestically. This and the discussion provided there serve to clarify the role that the subsidies to energy consumption play, and that the ownership of the claims to cash flows of firms and energy play. Appendix B.2 derives the linearized version of the heterogeneous-household model and the IS curve and NKPC. Appendix B.3 provides the proof for the proposition presented in Appendix B.1. Appendix B.4 provides the proof for Proposition 2 in the main text.

The current appendix discusses determinacy conditions for the full model of Section 2, that is, the two-household counterpart in which energy may be owned domestically or abroad, however, without considering energy in consumption.

B.1 Additional proposition – energy is owned domestically

The main text provides a proposition for the case that energy is owned by Foreign, $\iota = 0$ (Proposition 2 of the main text). Here, for completeness, we also provide the case that energy is owned domestically, $\iota = 1$ (not reported in the main text):

Proposition B.1. ***Energy owned in Home**. Consider the model of Section 2 and apply the assumptions listed in Section 3.1 for the heterogeneous-household case; however, do not restrict σ and suppose that energy is fully owned domestically, $\iota = 1$. In this case, $\widetilde{\kappa} = \frac{\varepsilon}{\psi}\left(\sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}\right) > 0$. So that having a fixed energy factor ($\alpha > 0$) makes inflation unambiguously more responsive to output. We have the following special cases for $\widetilde{\sigma}$.*

1. *$\nu = \vartheta = 0$: hand-to-mouth households only receive labor income. Then*

$$\widetilde{\sigma} = \sigma \cdot \frac{1 - \lambda(1+\varphi)}{1-\lambda}.$$

2. *$\nu = \vartheta$: hand-to-mouth households receive an equal share of profit income and energy income. Then*

$$\widetilde{\sigma} = \sigma \cdot \frac{1 - \lambda(1+\varphi - \varphi \cdot \vartheta/\lambda)}{1-\lambda}.$$

3. *$\nu = 0, \vartheta \geq 0$: hand to mouth households receive no profit income, but may participate in energy revenues. Then*

$$\widetilde{\sigma} = \sigma \cdot \frac{1 - \lambda\left(1 + \varphi + \frac{\varphi\alpha}{(1-\alpha)\sigma+\varphi}\frac{\vartheta}{\lambda}\left(\sigma + \frac{\varphi}{1-\alpha} + \frac{1/\theta}{1-\alpha}\right)\right)}{1-\lambda}.$$

Thus, $\widetilde{\sigma}^{(3)} < \widetilde{\sigma}^{(1)} < \widetilde{\sigma}^{(2)}$, where the subscript refers to $\widetilde{\sigma}$ in the respective case.

Proof. The proof is provided in Section B.3. □

Proposition B.1 combined with Proposition 1 has three implications. First, since $\widetilde{\kappa}$ is increasing in α, looking from the production side alone, the presence of the fixed factor *decreases* the risk of indeterminacy. Second, whether the demand side increases the risk of indeterminacy depends on the distribution of income. When hand-to-mouth households draw only labor income, the IS-curve looks exactly as in Bilbiie (2021) (our case 1). If hand-to-mouth households share equally in profit and energy income (case 2), indeterminacy gets less likely than in case 1. The reason is that profits are more *counter*cyclical than energy income is *pro*cyclical. If hand-to-mouth households do not share in profits but in the revenue from energy (case 3), instead, the demand side unambiguously amplifies the risk of indeterminacy. The reason for the latter is that energy income is procyclical. Third, the second observation is important because it illustrates the role of energy subsidies for indeterminacy. Namely, for the logic of the second implication, it does not matter whether hand-to-mouth households own energy out-right, or whether fiscal policy generates income streams that make hand-to-mouth households share in the energy revenues. In this sense, energy subsidies for households amplify the risk of indeterminacy.

B.2 Steady state and linearized equilibrium conditions

This section reports the steady and the linearized equilibrium conditions for the full model of Section 2 under the assumptions spelled out in Section 3.1 for the heterogeneous-household case, however, without restricting σ and or ι for now. So as to allow for potentially zero steady-state values of some variables, in this appendix, we linearize rather than log-linearizing; so that, in terms of notation, $\widehat{X}_t = (X_t - X)$.

B.2.1 Steady state

By assumption, we normalize $\xi_E = 1$ and choose χ such that $N = 1$, $E = 1$ and $Y_G = 1$. With an efficient steady state, $\Lambda = 1$, hence, $w = (1-\alpha)$ and $p_E = \alpha$. Note that $Y_G = C + X_G$ and $X_G = (1-\iota)p_E$, where ι is the share of energy owned by households in Home. Hence, $C = Y_G - X_G = 1 - (1-\iota)\alpha = 1 - \alpha + \iota\alpha$. Last, we target a steady state with equal consumption by all households. Hence, $C_H = C_S = C$ and $N_H = N_S = N$. This also implies that $R = 1/\beta$.

B.2.2 Linearized equilibrium conditions

On the household side, the Euler equation of savers is

$$C_S^{-1}\widehat{C}_{S,t} = \mathbb{E}_t C_S^{-1}\widehat{C}_{S,t+1} - \frac{1}{\sigma}\left[R^{-1}\widehat{R}_t - \mathbb{E}_t\widehat{\Pi}_{t+1}\right]. \tag{23}$$

Labor supply curves of spenders and savers are, for $i \in \{H, S\}$,

$$w^{-1}\widehat{w}_t = \sigma C_i^{-1}\widehat{C}_{i,t} + \varphi N_i^{-1}\widehat{N}_{i,t}. \tag{24}$$

The budget constraints of spenders and savers are, for $i \in \{H, S\}$,

$$\widehat{C}_{i,t} = wN_i\left(w^{-1}\widehat{w}_t + N_i^{-1}\widehat{N}_{i,t}\right) + \widehat{T}_{i,t}, \tag{25}$$

Lump-sum incomes of spenders and savers, using $\xi_E = 1$, are

$$\lambda\widehat{T}_{H,t} = \nu\widehat{D}_t + \iota\vartheta\widehat{p}_{E,t}, \qquad \text{and} \qquad (1-\lambda)\widehat{T}_{S,t} = (1-\nu)\widehat{D}_t + \iota(1-\vartheta)\widehat{p}_{E,t}. \tag{26}$$

And aggregation of household-level variables is given by

$$\widehat{N}_t = \lambda\widehat{N}_{H,t} + (1-\lambda)\widehat{N}_{S,t}, \tag{27}$$
$$\widehat{C}_t = \lambda\widehat{C}_{H,t} + (1-\lambda)\widehat{C}_{S,t}. \tag{28}$$

On the firm side, the production function, using $E_t = E = 1$ and $Y_G = N = 1$, is

$$Y_G^{-1}\widehat{Y}_{G,t} = (1-\alpha)N^{-1}\widehat{N}_t. \tag{29}$$

The Phillips curve is

$$\widehat{\Pi}_t = \beta\mathbb{E}_t\widehat{\Pi}_{t+1} + \frac{\varepsilon}{\psi}\widehat{\Lambda}_t. \tag{30}$$

Marginal costs, using $E_t = E$, are determined by

$$w^{-1}\widehat{w}_t = \Lambda^{-1}\widehat{\Lambda}_t + \frac{1}{\theta}\left(Y_G^{-1}\widehat{Y}_{G,t} - N^{-1}\widehat{N}_t\right), \tag{31}$$
$$p_E^{-1}\widehat{p}_{E,t} = \Lambda^{-1}\widehat{\Lambda}_t + \frac{1}{\theta}Y_G^{-1}\widehat{Y}_{G,t}. \tag{32}$$

And firms' profits, using $E_t = E = 1$, are

$$\widehat{D}_t = \widehat{Y}_{G,t} - wN\left(w^{-1}\widehat{w}_t + N^{-1}\widehat{N}_t\right) - \widehat{p}_{E,t}. \tag{33}$$

Monetary policy is

$$R^{-1}\widehat{R}_t = \phi_\Pi\widehat{\Pi}_t. \tag{34}$$

Foreign demand for exports, using $\xi_E = 1$, is

$$\widehat{X}_{G,t} = (1-\iota)\widehat{p}_{E,t}. \tag{35}$$

The goods market clears with

$$\widehat{Y}_{G,t} = \widehat{C}_t + \widehat{X}_{G,t}. \tag{36}$$

B.2.3 Consolidating to NKPC and IS curve

Proposition 2 in the main text and Proposition B.1 in the appendix rely on a three-equation representation of the economy: The New-Keynesian Phillips curve, the IS equation and the monetary policy rule. This section derives the former two for the model with heterogeneous households and a flexible energy-ownership structure.

Goods market clearing. Using foreign demand (35), (36) can be rewritten as

$$\widehat{Y}_{G,t} = \widehat{C}_t + (1-\iota)\widehat{p}_{E,t}. \tag{37}$$

Marginal costs. Using household symmetry, the labor supply curves (24) can be aggregated to $w^{-1}\widehat{w}_t = \sigma C^{-1}\widehat{C}_t + \varphi N^{-1}\widehat{N}_t$. Combining this with firms' labor demand (31), energy demand (32), goods market clearing (37), and the production function (29) yields

$$\Lambda^{-1}\widehat{\Lambda}_t = \Gamma_\Lambda Y_G^{-1}\widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_\Lambda = \frac{\sigma + \varphi - \frac{\alpha}{\theta}(\sigma-1) + \iota\left[\frac{\alpha}{1-\alpha}\left(\varphi + \frac{\alpha}{\theta}\right) + \sigma\frac{\alpha}{\theta}\right]}{1 + (1-\iota)\alpha(\sigma-1)}. \tag{38}$$

Notably, household heterogeneity does not affect elasticity Γ_Λ, see Appendix A.

Phillips curve. Combining the Phillips curve (30) with (38) yields a usual representation of the New Keynesian Phillips curve

$$\widehat{\Pi}_t = \beta\mathbb{E}_t\widehat{\Pi}_{t+1} + \widetilde{\kappa}\widehat{Y}_{G,t} \qquad \text{with} \qquad \widetilde{\kappa} = \frac{\varepsilon}{\psi}\Gamma_\Lambda. \tag{39}$$

Since Γ_Λ is not affected by household heterogeneity, neither is $\widetilde{\kappa}$—as stated above.

Energy prices and wages. Firms' labor and energy demand (31) and (32) can be combined with firms' production function (29) and (38) to yield

$$w^{-1}\widehat{w}_t = \Gamma_w Y_G^{-1}\widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_w = \Gamma_\Lambda + \frac{1}{\theta} - \frac{1}{\theta(1-\alpha)}, \tag{40}$$

$$p_E^{-1}\widehat{p}_{E,t} = \Gamma_{p_E} Y_G^{-1}\widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_{p_E} = \Gamma_\Lambda + \frac{1}{\theta}. \tag{41}$$

Hence, $\frac{1}{\theta(1-\alpha)} > 0$ measures the excess elasticity of energy prices with respect to production, relative to the elasticity of wages with respect to production. This excess elasticity is increasing in α and decreasing in θ.

Firms' dividends. Using (40) and (41) and firms' production function (29), firms' dividends (33) can be expressed in terms of production as

$$\widehat{D}_t = \Gamma_D Y_G^{-1} \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_D = -\Gamma_\Lambda. \tag{42}$$

Spenders' lump-sum income. Using (42) and (41), spenders' lump-sum income (26) can be expressed in terms of production as

$$\widehat{T}_{H,t} = \Gamma_{T_H} Y_G^{-1} \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_{T_H} = \frac{\nu}{\lambda}\Gamma_D + \iota\frac{\vartheta}{\lambda}\alpha\Gamma_{p_E}. \tag{43}$$

Spenders' total income. Using spenders' labor supply curve (24), their budget (25) as well as (40) and (43), spenders' income is given by

$$C_H^{-1}\widehat{C}_{H,t} = \Gamma_{C_H} Y_G^{-1} \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_{C_H} = \frac{(1+\varphi)\Gamma_w}{\sigma + \varphi + \iota\frac{\alpha}{1-\alpha}\varphi} + \frac{\varphi/(1-\alpha)\Gamma_{T_H}}{\sigma + \varphi + \iota\frac{\alpha}{1-\alpha}\varphi}, \tag{44}$$

with Γ_Λ defined in (38), Γ_w defined in (40), Γ_{p_E} defined in (41), Γ_D defined in (42), and Γ_{T_H} defined in (43).

Aggregate demand. Recall that aggregate demand is given by (37), so that, using (41), aggregate domestic consumption is given by

$$C^{-1}\widehat{C}_t = \Gamma_C Y_G^{-1} \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_C = \frac{1-(1-\iota)\alpha\Gamma_{p_E}}{1-\alpha+\iota\alpha}. \tag{45}$$

Savers' consumption. From aggregate consumption (45) and aggregation (28), using symmetry of households ($C_S = C_H = C$), we can derive savers' consumption as

$$C_S^{-1}\widehat{C}_{S,t} = \Gamma_{C_S} Y_G^{-1} \widehat{Y}_{G,t} \qquad \text{with} \qquad \Gamma_{C_S} = \frac{\Gamma_C - \lambda\Gamma_{C_H}}{1-\lambda}. \tag{46}$$

Note that the sign of Γ_{C_S} is fully determined by $\Gamma_C - \lambda\Gamma_{C_H}$.

IS curve. Combining savers' Euler equation (23) with (46), and using the monetary policy rule (34), yields the IS curve

$$\widehat{Y}_{G,t} = \mathbb{E}_t\widehat{Y}_{G,t+1} - \frac{1}{\widetilde{\sigma}}\left(\phi_\Pi\widehat{\Pi}_t - \mathbb{E}_t\widehat{\Pi}_{t+1}\right) \qquad \text{with} \qquad \widetilde{\sigma} = \sigma\Gamma_{C_S}. \tag{47}$$

Note that $\widetilde{\sigma} > 0$ if and only if $\Gamma_{C_S} > 0$. This is the case if and only if $\Gamma_C - \lambda\Gamma_{C_H} > 0$.

B.3 The closed economy

Proposition B.1 focuses on an economy with heterogeneous households in which domestic households fully own the energy that the economy uses ($\iota = 1$, the "closed" economy). This economy is the focus of the current section. We first document how assuming $\iota = 1$ simplifies the expressions above and, then, present the proof of the proposition.

B.3.1 Simplified expressions for the closed economy

Firms' side. (38) simplifies to $\Gamma_\Lambda = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha} > 0$. Hence, marginal costs unambiguously increase in output. This means that $\widetilde{\kappa} > 0$. This means that the Phillips curve never inverts in the closed economy. (40) and (41) are $\Gamma_w = \sigma + \frac{\varphi}{1-\alpha}$ and $\Gamma_{p_E} = \sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}$.

Spenders' total income. (44) simplifies to

$$\Gamma_{C_H} = 1 + \varphi + \frac{\varphi}{(1-\alpha)\sigma + \varphi}\left[-\frac{\nu}{\lambda}\left(\sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha/\theta}{1-\alpha}\right) + \frac{\vartheta}{\lambda}\alpha\left(\sigma + \frac{\varphi}{1-\alpha} + \frac{1/\theta}{1-\alpha}\right)\right]. \quad (48)$$

Ownership of dividends (ν) renders spenders' income (ceteris paribus) more countercyclical, while ownership of energy revenues (ϑ) renders spenders' income (ceteris paribus) more procyclical. The effect of owning dividends is stronger than the effect of owning energy revenues, since ϑ is premultiplied by $\alpha \in [0, 1)$ in the second term of Γ_{C_H}.

Aggregate demand. Consumption and production coincide, $\Gamma_C = 1$.

IS Curve. From (46), the IS curve inverts if $\Gamma_{C_S} < 0$, i.e., when $\lambda\Gamma_{C_H} > 1$. This is as in the standard TANK model, with the innovation that, with scare factor inputs, Γ_{C_H} is affected by the ownership of energy revenues and dividends.

B.3.2 Proof of Proposition B.1

Consider the assumptions of Proposition B.1 and recall the terms derived in Appendix B.3.1. Note that since $\widetilde{\kappa} > 0$ in the closed economy, the same two cases for (in)determinacy arise as in Proposition 1 of the main text; what is more, as before, indeterminacy can arise only in Case 2) of Proposition 1. Proposition B.1 considers three cases:

Case 1: If spenders neither receive dividends nor energy revenues, then (48) simplifies to $\Gamma_{C_H} = 1 + \varphi$, in fact, recovering the same elasticity as in Bilbiie (2008).[19]

[19] When there is no energy in the economy ($\alpha = 0$), then $\Gamma_{C_H} = 1 + \varphi - \varphi\frac{\nu}{\lambda}$, in fact, recovering the case of a standard TANK model with dividends potentially owned by spenders. Note that, for this case, if spenders do not own dividends ($\nu = 0$), then $\Gamma_{C_H} = 1 + \varphi$.

Case 2: If spenders own both dividends and energy revenues to the same proportion ($\vartheta = \nu$), then from (48)

$$\Gamma_{C_H} = 1 + \varphi - \varphi \frac{\vartheta}{\lambda} < 1 + \varphi,$$

thus, spenders' income is less procyclical than in the benchmark case.
Case 3: If spenders own only energy ($\nu = 0$, $\vartheta \geq 0$), their income becomes more procyclical. From (48)

$$\Gamma_{C_H} = 1 + \varphi + \frac{\varphi\alpha}{(1-\alpha)\sigma + \varphi} \frac{\vartheta}{\lambda} \left(\sigma + \frac{\varphi}{1-\alpha} + \frac{1/\theta}{1-\alpha} \right) > 1 + \varphi.$$

This, along with $\Gamma_C = 1$ from market clearing and the expressions for Γ_{C_S} and for $\widetilde{\sigma}$ given in equations (46) and (47), respectively, establishes the statements in the proposition. □

B.4 The open economy

Proposition 2 focuses on an economy with heterogeneous households in which all the energy is held abroad ($\iota = 0$, the "open" economy). This economy is the focus of the current appendix. We first document how assuming $\iota = 0$ simplifies the expressions above and, then, present the proof of the proposition.

B.4.1 Simplified expressions for the open economy

Firms' side. (38) simplifies to $\Gamma_\Lambda = \frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)}$. This means that marginal costs do not necessarily increase in output. $\Gamma_\Lambda > 0$ holds if and only if $1 - \alpha/\theta > -1/\sigma(\varphi + \alpha/\theta)$. For a discussion of the sign of Γ_Λ, see Appendix A. Note that $\widetilde{\kappa} > 0$ if and only if $\Gamma_\Lambda > 0$. Hence, contrary to the closed economy in the open economy not only the IS curve can be inverted, but also the Phillips curve.

Spenders' total income. (44) simplifies to

$$\Gamma_{C_H} = \frac{1+\varphi}{\sigma+\varphi} \left(\frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)} - \frac{\alpha/\theta}{1-\alpha} \right) - \frac{\nu}{\lambda} \times \frac{\varphi}{\sigma+\varphi} \frac{1}{1-\alpha} \frac{\sigma+\varphi-\alpha/\theta(\sigma-1)}{1+\alpha(\sigma-1)}. \quad (49)$$

Ownership of dividends (ν) can make spenders' income (ceteris paribus) more or less procyclical, depending on the sign of $\sigma + \varphi - \alpha/\theta(\sigma - 1)$, see Γ_Λ. Considering the case where $\Gamma_\Lambda > 0$, their income becomes less procyclical the larger the profit share.

Aggregate demand. In the open economy, consumption can potentially *fall* even though production rises. This is because consumption has to equal value added. When energy prices increase, the value-added share of production falls. Γ_C need not equal unity, see equation (45).

IS Curve. From (46), the IS curve inverts if $\Gamma_{C_S} < 0$, i.e., when $\lambda\Gamma_{C_H} > \Gamma_C$. The difference to the closed economy is that here, energy also affects Γ_C.

B.4.2 Proof of Proposition 2

Consider the assumptions of Proposition 2 and recall the terms derived in Appendix B.4.1. Proposition 2 assumes that $\sigma = 1$ (no wealth effect on labor supply). With this, from (38), $\Gamma_\Lambda = 1 + \varphi$, so that $\widetilde{\kappa} > 0$. Hence, once more, there are the same two cases regarding (in)determinacy as discussed in Proposition 1. What is more, for given φ, the other parameters affect (in)determinacy only through $\widetilde{\sigma}$.
Expression (49) simplifies to

$$\Gamma_{C_H} = 1 + \varphi - \frac{\alpha/\theta}{1-\alpha} - \frac{\nu}{\lambda}\frac{\varphi}{1-\alpha}.$$

Next, with (41) and (45), $\Gamma_C = \frac{1-\alpha[1+\varphi+1/\theta]}{1-\alpha}$. Combining the expression for Γ_{C_S} in (46) and for $\widetilde{\sigma}$ in (47), we have (with $\sigma = 1$)

$$\widetilde{\sigma} = (1-\lambda)^{-1}\left[\frac{1-\alpha\,[1+\varphi+1/\theta]}{1-\alpha} - \lambda\left(1+\varphi-\frac{\alpha/\theta}{1-\alpha}-\frac{\nu}{\lambda}\frac{\varphi}{1-\alpha}\right)\right].$$

The derivatives are given by $\partial\widetilde{\sigma}/\partial\alpha = (1-\lambda)^{-1}\left[-\frac{\varphi(1-\nu)+1/\theta(1-\lambda)}{(1-\alpha)^2}\right] < 0$ and $\partial^2\widetilde{\sigma}/\partial\alpha\partial\lambda = -\frac{\varphi(1-\nu)}{(1-\alpha)^2}\frac{1}{(1-\lambda)^2} < 0$. This proves the Proposition 2. $\square$